* For the following, we've some substitutions:-

$\mu \rightarrow \bar{x}$

$\sigma^2 \rightarrow s^2$

$\sigma \rightarrow s$

$p \rightarrow \hat{p}$

where $\bar{x} = \dfrac{\sum x}{n}$

* Sampling distribution of $\bar{x}$

→ $\bar{x} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ (unbiased estimator)

→ $Z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

→ for unknown population variance:-

t-distribution is used

→ $t = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$ & df $(n-1)$

$\dfrac{\sigma}{n} \rightarrow Z$

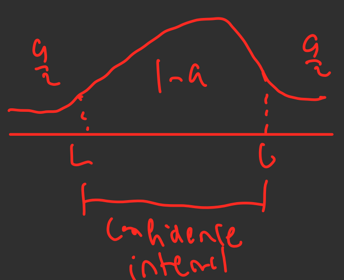
$\dfrac{s}{n} \rightarrow t < 30$, $z \geq 30$

* Interval estimation:

(L, U) is our confidence interval

1- alpha is confidence level

$P(L < \theta < U) = 1-\alpha$



* for the confidence Interval

(L, U)

$(\bar{X} - \epsilon, \bar{X} + \epsilon)$

$$\left(\bar{X} - Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right)$$

→ $\epsilon = Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$ (margin Error)

→ S.E $= \frac{\sigma}{\sqrt{n}}$

→ Critical value: $Z_{\frac{\alpha}{2}}$

→ Length of interval (width): $L = 2\epsilon$

* Notes:-

1) If pop. var is known, find C.I for μ by:

$$\left(\bar{X} - Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}, \bar{X} + Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$$

2) If pop var is unknown,

$$\left(\bar{X} - t_{\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}, \bar{X} + t_{\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}\right)$$

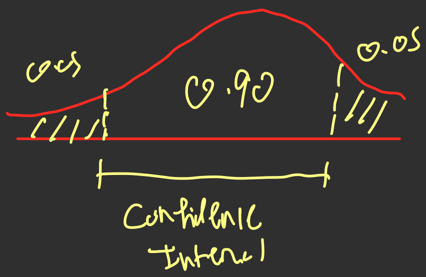
3) if $n \geq 30$ & σ^2 is unknown, use Z

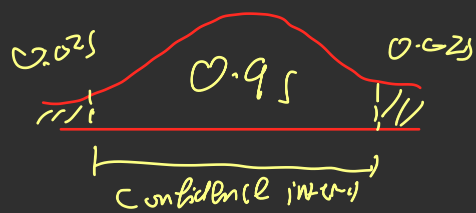
$$\left(\bar{X} - Z_{\frac{\alpha}{2}} \cdot \frac{S}{\sqrt{n}}, \bar{X} + Z_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}\right)$$

→ Notes:-

1) As n inc., E dec & C.I becomes narrower ie more precise

2) As C.L inc., E inc. so C.I inc. becomes wider





* For Determining Sample size (n):-

$$E = Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \quad \therefore \quad n = \left(Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{E} \right)^2$$

$$L = 2E$$

→ If you get a fraction, round up!!

(Don't be a bitch)

* Distribution of Sample Proportion ($\hat{p}$)

→ This is basically when we're talking about taking a sample of the population & studying it.

→ Here the proportion is p but sample is $\hat{p}$.

→ $\hat{p} \sim N\left(p, \frac{pq}{n}\right)$ only if $npq \geq 5$

→ $$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \sim N(0,1)$$

* for the C.I

→ $\hat{p} = \frac{x}{n}$

→ $\hat{p} = \frac{L+U}{2}$

→ (L, U)

$(\hat{p} - \epsilon, \hat{p} + \epsilon)$

→ $\left(\hat{p} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}, \ \hat{p} + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}\right)$

→ $\epsilon = z_{\frac{\alpha}{2}} \sqrt{\frac{\hat{p}\hat{q}}{n}}$ → $S.E = \sqrt{\frac{\hat{p}\hat{q}}{n}}$

* only if $n\hat{p}\hat{q} \geq 5$

* for finding sample size (n) :- for C.I

$E = Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$ $\therefore n = \left(\frac{Z_{\alpha/2}}{E}\right)^2 \cdot (\hat{p} \cdot \hat{q})$

$\hat{p}$ either
- → given in question
- → if not given it's $\hat{p} = 0.5$

Length (L) = 2 E

Summary Book Questions:-

Q.1) if n is Large, Sample mean is approximately normal with μ & σ^2

$\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

Q.2) $\bar{x} = 8$ $\sigma = 4$ $n = 30$

1) $P(\bar{X} < 7)$

$P\left(Z < \frac{7-8}{4/\sqrt{30}}\right)$

$P(Z < -1.37)$

$P(Z > 1.37)$ col. B

$\boxed{0.0853}$

2) $P(\bar{X} > 7)$

$P\left(Z > \frac{7-8}{4/\sqrt{30}}\right)$

$P(Z > -1.37)$

$P(Z < 1.37)$ col. A

$\boxed{0.9147}$

3) $P(7 < \bar{X} < 9)$

$P(\bar{X} < 9) - P(\bar{X} < 7)$

$P\left(Z < \frac{9-8}{4/\sqrt{30}}\right) - P\left(Z < \frac{7-8}{4/\sqrt{30}}\right)$

$P(Z < 1.37) - P(Z < -1.37)$

$P(Z < 1.37) - P(Z > 1.37)$ col. B

$= 0.9147 - 0.0853$

$= \boxed{0.8294}$

Q.3) $\bar{X} = 5.4$ $\sigma = 2.28$ $n = 36$

$P(\bar{X} \leq 4.8)$

$P\left(Z \leq \frac{4.8 - 5.4}{2.28/\sqrt{36}}\right)$

$P(Z < -1.05)$

$= P(Z > 1.05)$ col. B $\Rightarrow \boxed{0.1469}$

Q.4) $\bar{X} = 120$ $\sigma = 10$ $n = 25$

$P(120 < \bar{X} < 123)$

$P(\bar{X} < 123) - P(\bar{X} < 120)$

$P\left(Z < \frac{123-120}{10/\sqrt{25}}\right) - P\left(Z < \frac{120-120}{10/\sqrt{25}}\right) \Rightarrow P(Z < 1.5) - P(Z < 0)$

$0.9332 - 0.5$

$= 0.4332$

Q.5) $\bar{x} = 200 \quad \sigma = 15 \quad n = 30$

$P(\sum x_i \geq 6060)$ $\qquad \bar{x} = \frac{\sum x_i}{n}$

$P\left(\frac{\sum x_i}{n} \geq \frac{6060}{n}\right)$

$P\left(\bar{x} \geq \frac{6060}{30}\right)$

$P(\bar{x} \geq 202) \rightarrow P\left(Z \geq \frac{202-200}{15/\sqrt{30}}\right)$

$P(Z \geq 0.73)$ col.8 $\rightarrow 0.2327$

Q.6) $\bar{x} = 120 \quad \sigma = 10 \quad n = 25$ $\qquad$ 30th percentile

$x \sim N(120, 10^2) \rightarrow \bar{x} \sim N\left(120, \frac{10^2}{25}\right)$

$P(\bar{x} \leq P_{30}) = 0.3$

$P\left(Z \leq \frac{P_{30} - 120}{10/\sqrt{25}}\right) = -0.52$

$\frac{P_{30} - 120}{10/\sqrt{25}} = -0.52 \rightarrow \boxed{\therefore P_{30} = 118.96}$

Q.1)

1) $P(t > 2.35)$ df 3

$1 - P(t < 2.35)$ df(3)

$1 - 0.95 = 0.05$

2) $P(t < P_{10}) = 0.1$ df(60)

$\quad \quad 0.9$

$P_{10} = 1.296$ but

-1.296

3) $P(t > 1\%)$

$P(t < 99\%) = 0.99 \; df(16)$

$P(t < 0.99) = 0.99 \; df\ 16$

$\boxed{P_{99} = t\ 2.583}$

4) $P(t < 97.5\%) = 0.975 \; df(7)$

$P(t < 0.975) = 0.975 \; df(7)$

$\boxed{P_{0.975} = 2.365}$

Q.1) $\bar{x} = 3 \quad n = 10 \quad \sigma = 2$

1) $P(\bar{x} < 4.16)$

$P\left(t < \frac{4.16 - 3}{2/\sqrt{10}}\right)$

$P(t < 1.83) \; df(9)$

$= \boxed{0.95}$

2) $P(\bar{x} < P_{90}) = 0.90 \; df(9)$

$P\left(t < \frac{P_{90} - 3}{2/\sqrt{10}}\right) = 1.383$

$\frac{P_{90} - 3}{2/\sqrt{10}} = 1.383$

$\boxed{\therefore P_{90} = 3.87}$

Q.3) $n = 16 \quad \bar{x} = 20 \quad s.d = 5$

$P(\bar{x} > c) = 0.05$

$P\left(t > \frac{c - 20}{5/\sqrt{16}}\right) = 0.05$

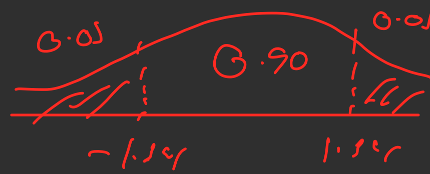
$P\left(t < \frac{c - 20}{5/\sqrt{16}}\right) = 0.95 \; df(15)$

$\frac{c - 20}{5/\sqrt{16}} = 1.753$

$\boxed{\therefore c = 22.19}$

Q.4) df(15) 0.90 [illegible]

so 0.9 & df(15) -1.341



Q.5) $\bar{x} = 15$ $n = 25$ $sd = 3$

$P(\bar{x} < 16)$

$P\left(t < \frac{16-15}{3/\sqrt{25}}\right) \rightarrow P(t < 1.7)$ df = 24

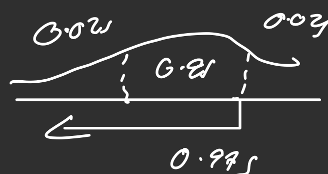
$= 0.95$

Q.1) $n = 20$ $\bar{x} = 22.9$ $\sigma = 1.5$

1) $\bar{x} = 22.9$

2) C.I 95%

$(\bar{x} - E, \bar{x} + E)$



$E = Z_{\frac{\alpha}{2}} \cdot \frac{s.d}{\sqrt{n}}$

$Z(0.975) = 1.96$

$E = 1.96 \cdot \frac{1.5}{\sqrt{20}}$

$E = \text{Ans}$

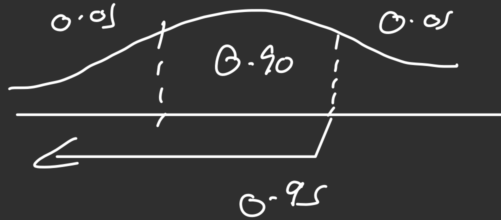
$\bar{x} \pm \text{Ans}$

$(22.24, 23.56)$

for a check :- $\bar{x} = \frac{22.24 + 23.56}{2} = 22.9$

Q.4) $n = 16$ $\bar{x} = 220$ $\sigma = 25$ $a = 0.90$

$(\bar{x} - E, \bar{x} + E)$



$E = t_{\frac{a}{2}} \cdot \frac{s.d}{\sqrt{n}}$

$E = 1.753 \cdot \frac{25}{\sqrt{16}}$

$t_{0.95}(15) = 1.753$

$\underline{E = Ans}$

$\bar{x} \pm Ans = (209.04, 230.96)$

* check

$\frac{209.04 + 230.96}{2}$

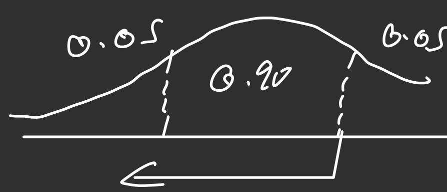
$= 220 = \bar{x}$ ✓

Q.5) $n = 48$ $\bar{x} = 45.3$ $s.d = 3.8$

$a = 0.90$

C.I (L, U)

$(\bar{x} - E, \bar{x} + E)$



$E = Z_{\frac{a}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

$Z_{0.95}$

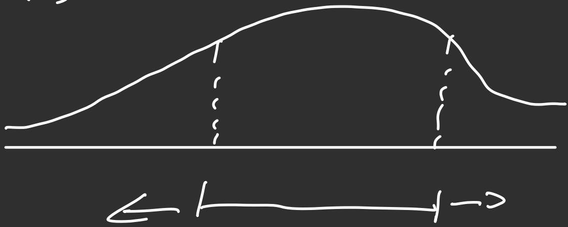
1.645

$E = 1.645 \cdot \frac{3.8}{\sqrt{48}}$

$E = Ans$

$\bar{x} \pm Ans = (44.4, 46.2)$

Q-4) What inc. C.I width ?



more n, reduce it

more variability ✓

more C.L ✓

Less n ✓

Less sig level ✓

Q-5) (26.2, 30.1)

$(\bar{x} - E, \bar{x} + E)$

$$\bar{x} = \frac{26.2 + 30.1}{2}$$

$$\bar{x} = 28.15$$

for E:

$26.2 = \bar{x} - E$ $\quad$ $30.1 = \bar{x} + E$

$E = \bar{x} - 26.2$ $\quad$ $30.1 - \bar{x} = E$

$E = 1.95$ $\quad$ $E = 1.95$

Q-6) (22, 35) $\quad a = 0.95, n = 36$

$$\bar{x} = \frac{22 + 35}{2} = 28$$

1.96

s.d = ?

$22 = \bar{x} - Z_{\frac{a}{2}} \cdot \frac{\sigma}{\sqrt{n}}$ $\quad$ or $\quad$ $35 = \bar{x} + Z_{\frac{a}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

$22 = 28 - 1.96 \cdot \frac{\sigma}{\sqrt{36}}$ $\quad$ $35 = 28 + 1.96 \cdot \frac{\sigma}{\sqrt{36}}$

$\therefore \sigma = 18.37$ $\quad$ $\therefore \sigma = 18.37$

Q.7) $(25, 35)$ $CL = 0.95$ $\alpha = 0.05$, $n = 10$

$$\bar{x} = \frac{25+35}{2} = 30$$

$t_{0.975}(9)$

$= 2.262$

$$25 = \bar{x} - t_{0.975}(9) \cdot \frac{\sigma}{\sqrt{n}}$$

$$25 = 30 - 2.262 \cdot \frac{\sigma}{\sqrt{10}}$$

$$\therefore \boxed{\sigma = 7}$$

→ At 98% C.I

$t_{0.99}(9)$

2.821

$\bar{x} = 30$

$\sigma = 7$

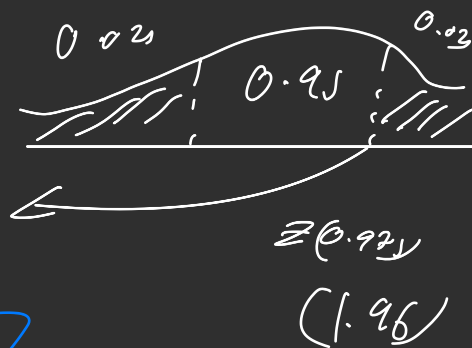
$$E = t_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$E = \underbrace{2.821 \cdot \frac{7}{\sqrt{10}}}_{} = Ans$$

$\bar{x} \pm \underset{(Ans)}{E} \Rightarrow 30 \pm Ans = (23.77, 36.23)$

Q.1) $\bar{X} = ?$ $s.d = 5$ $n = ?$ $C.L = 0.95$

$E = 1.5$



$E = Z_{(0.975)} \cdot \frac{\sigma}{\sqrt{n}}$

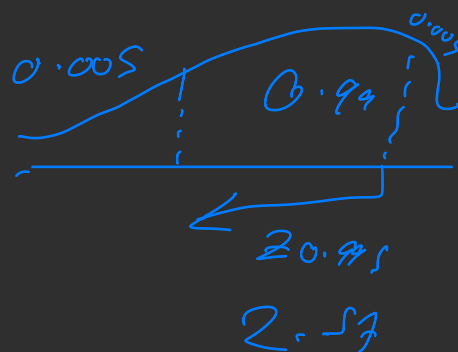
$\therefore n = \left(Z_{0.975} \cdot \frac{\sigma}{E}\right)^2$

$n = \left(1.96 \cdot \frac{5}{1.5}\right)^2$ $\boxed{\therefore n = 43}$

Q.2) $\sigma^2 = 144$

$L = 8 = 2E$

$\therefore E = 4$



$E = Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$

$n = \left(Z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{E}\right)^2 \Rightarrow \left(2.57 \cdot \frac{12}{4}\right)^2 = 60$

Q.1) $P_0 = 0.6$ $n = 100$

$P(\hat{p} < 0.63)$

$P\left(Z < \frac{0.63 - 0.6}{\sqrt{P_0 Q_0 / n}}\right) \rightarrow P\left(Z < \frac{0.63 - 0.6}{\sqrt{\frac{(0.6)(0.4)}{100}}}\right) = P(Z < 0.61)$

$= \underline{0.729}$

Q.2) $p=0.2$ $\hat{p}$, z

1) $\hat{p} \sim N\left(0.2, \frac{0.2\cdot 0.8}{200}\right)$

2) $P(\hat{p} \le .19)$

$$P\left(Z \le \frac{.19-0.2}{\sqrt{\frac{0.2\cdot 0.8}{200}}}\right) = P(Z \le -0.35) = P(Z \ge 0.35)$$

(0).B

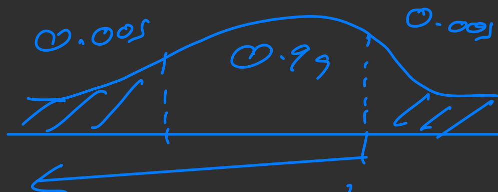
$\boxed{0.3632}$

Q.1) $n=100$, $x=40$, $C.I = 0.99$

$\hat{p} = \frac{x}{n} = \frac{40}{100} = 0.4$

(L, U)

$(\hat{p}-\epsilon, \hat{p}+\epsilon)$



$$\hat{p} - z_{0.995}\sqrt{\frac{\hat{p}\hat{q}}{n}}, \quad \hat{p} + z_{0.995}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$z_{0.995}$ (2.57)

$$0.4 \pm 2.57\sqrt{\frac{(0.4)(0.6)}{100}} = 0.4 \pm 0.126$$

Q.2)

1) $\hat{p} = \frac{x}{n} = \frac{620}{1000} = 0.62$

2) $S.E = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{0.62 \cdot 0.38}{1000}} = 0.0153$

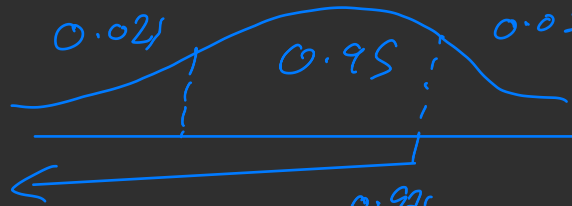
3) C.I $= 0.95$

(L, U)

$\hat{p} \pm z_{0.975} \cdot \sqrt{\frac{\hat{p}\,\hat{q}}{n}}$

$0.62 \pm 1.96 \cdot 0.0153$

$(0.59,\ 0.65)$



$z_{0.975}$
1.96

Q.1) C.I $= 0.95$, $\hat{p} = 0.5$

$L = 2E$

$0.14 = 2E$

$\therefore E = 0.07$

$z_{0.975}$
1.96

$n = \left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 \cdot \hat{p}\,\hat{q}$

$n = (1.96 \div 0.07)^2 \cdot (0.5)(0.5) = \boxed{196}$