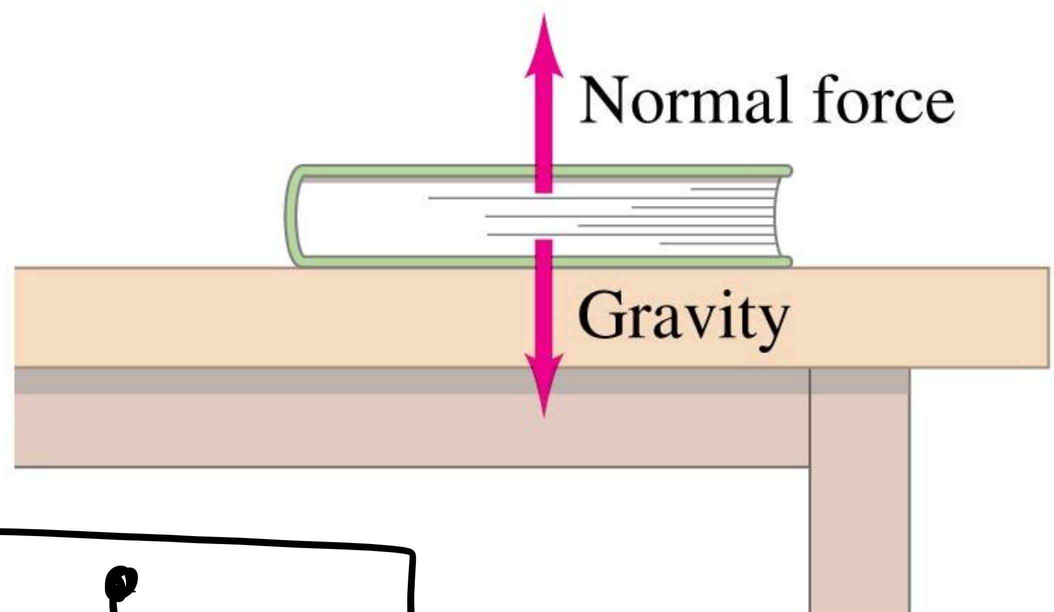
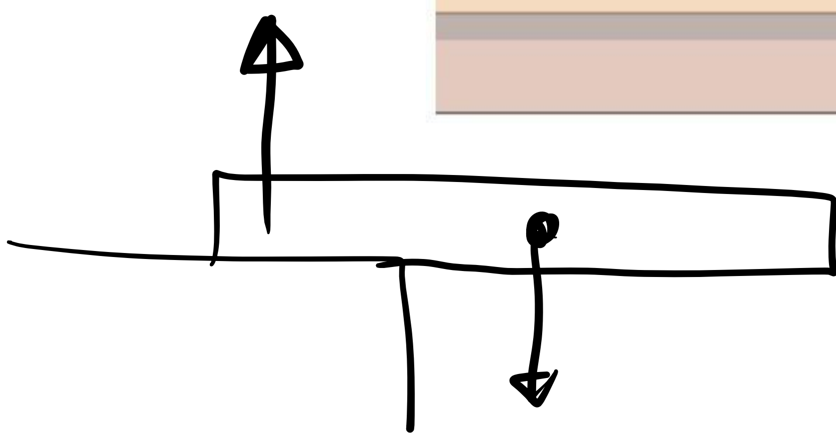
Chapter 9
Static Equilibrium;
Elasticity and Fracture

9-1 The Conditions for Equilibrium

$\sum F = 0$

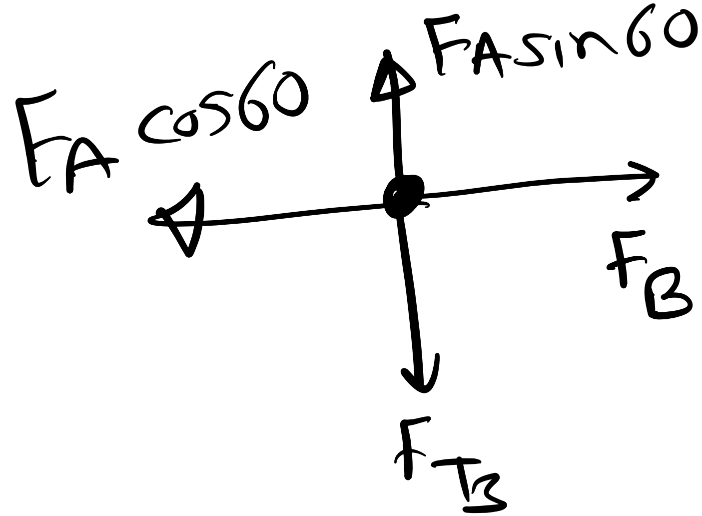
An object with forces acting on it, but that is not moving, is said to be in equilibrium.

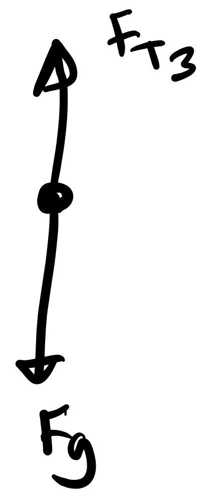


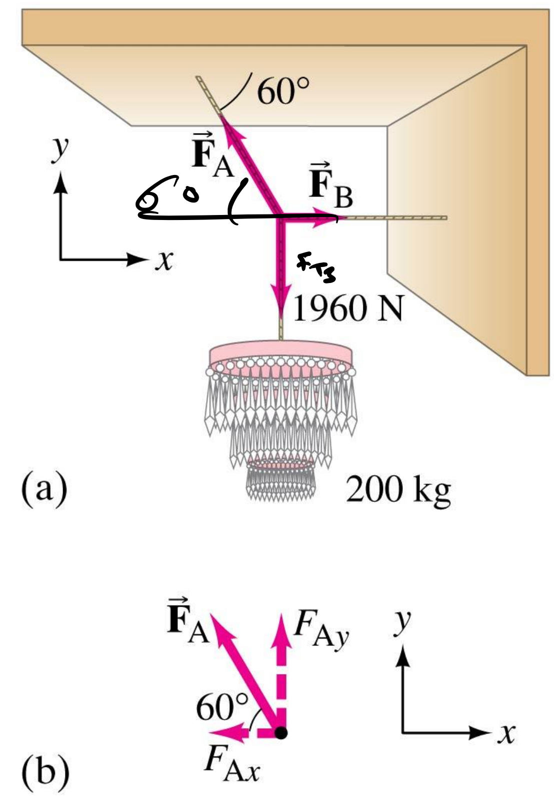


9-1 The Conditions for Equilibrium

The first condition for equilibrium is that the forces along each coordinate axis add to zero.







$$F_{T3} = 1960\,N$$

$$F_A \sin 60 = F_{T3} = 1960$$

$$F_A = \frac{1960}{\sin 60} = 2263\,N$$

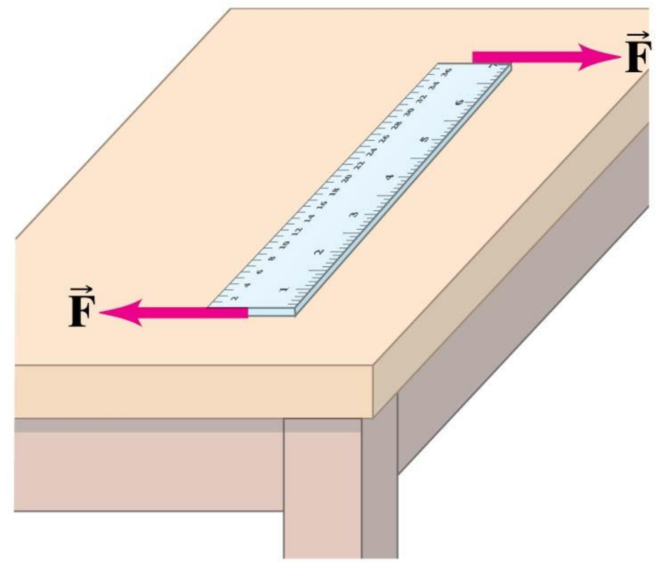
$$F_B = F_A \cos 60 = 1182\,N$$

9-1 The Conditions for Equilibrium

The second condition of equilibrium is that there be no torque around any axis; the choice of axis is arbitrary.

this is not is static equilibrium

because $\sum \tau \neq 0$



$\Rightarrow$ the 2nd condition must be satisfied

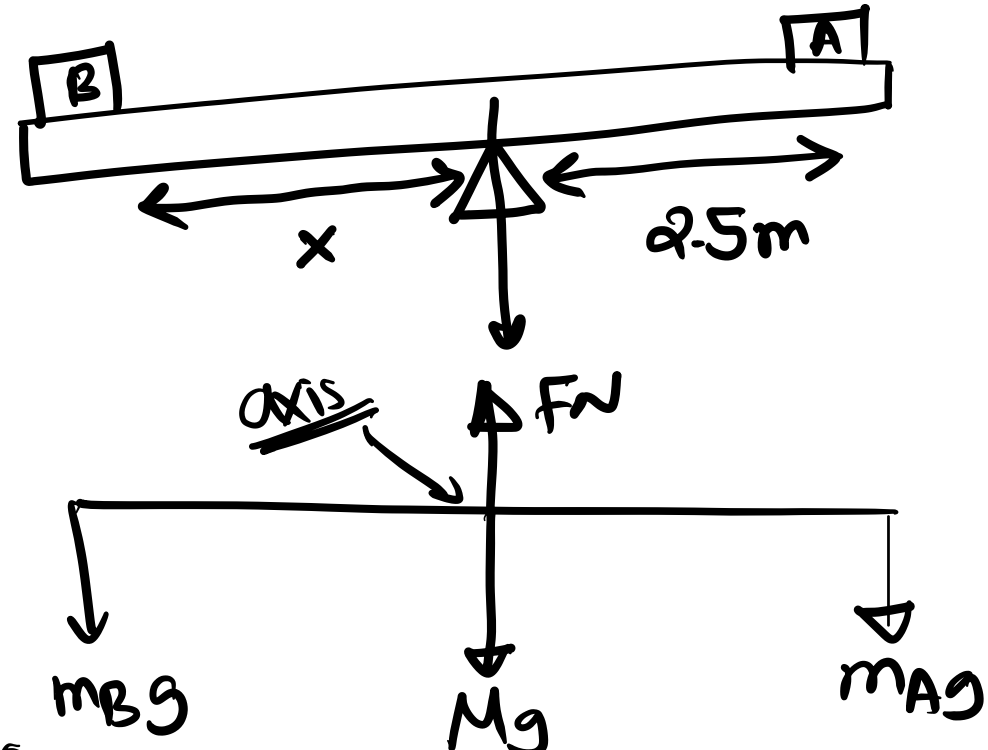
$\sum \tau = 0$

A board of mass M serves as a seesaw for two children. Child A has a mass of 30 kg and sits 2.5 m from the pivot point, P (his center of gravity is 2.5 m from the pivot). At what distance x from the pivot must child B, of mass 25 kg, place herself to balance the seesaw? Assume the board is uniform and centered over the pivot.

$$\Sigma F = 0$$

$$F_N - m_A g - Mg - m_B g = 0$$

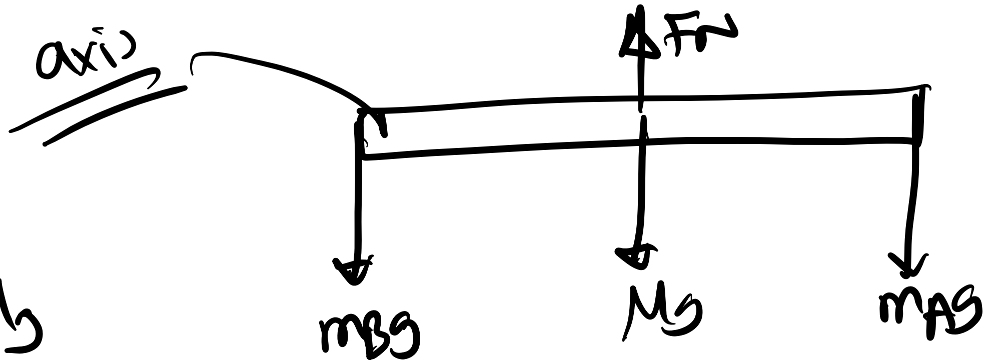
$$F_N = m_A g + Mg + m_B g$$



$$\Sigma \tau = 0$$

$$-m_A \cancel{g}(2.5) + m_B \cancel{g} x = 0$$

$$x = \frac{m_A(2.5)}{m_B} = \underline{3m}$$



$$F_N = m_A g + m_B g + Mg$$

$$\Sigma \tau = 0$$

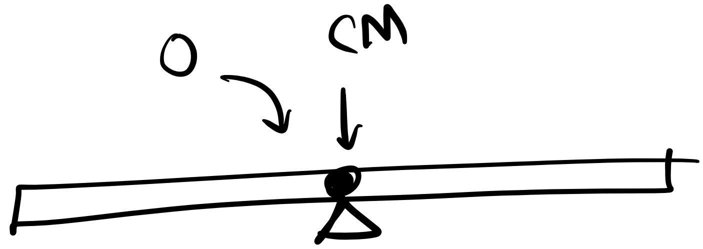
$$-m_A g(2.5+x) - Mgx + F_N x = 0$$

$$-m_A g(2.5) - \cancel{m_A g x} - \cancel{Mgx} + \cancel{m_A g x} + \cancel{Mgx} + m_B g x = 0$$

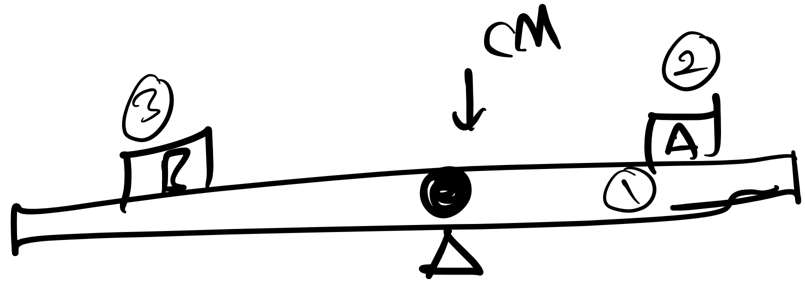
$$-m_A \cancel{g}(2.5) = -m_B \cancel{g} x$$

$$x = 3\,m$$

before



after



CM at origin

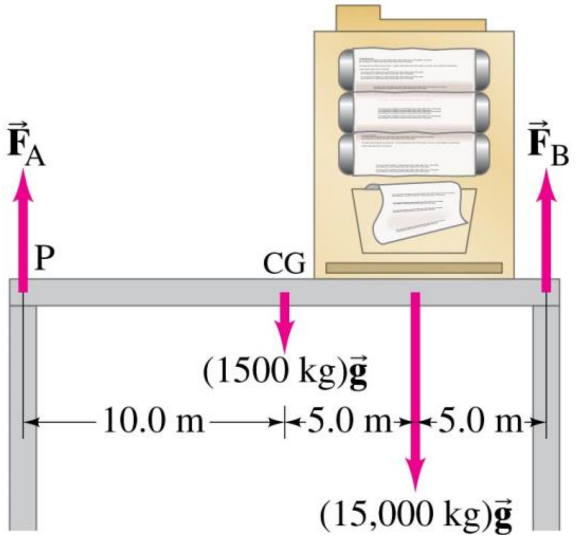
$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$0 = \frac{M(0) + 30(2.5) + 25(-x)}{m_A + M + m_B}$$

$$30(2.5) = 25x$$

$$x = 3\ m$$

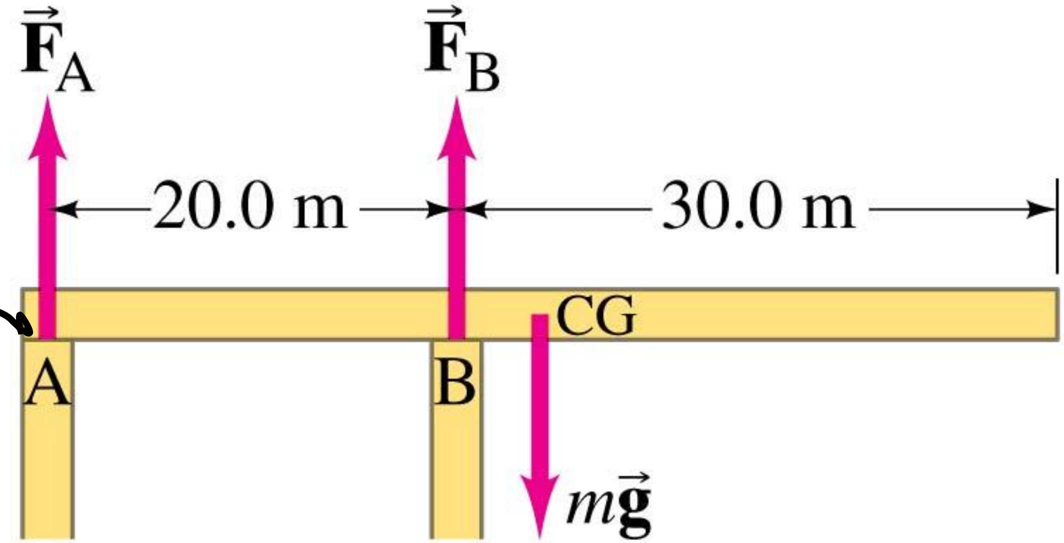
A uniform 1500-kg beam, 20.0 m long, supports a 15,000-kg printing press 5.0 m from the right support column. Calculate the force on each of the vertical support columns.



Find F_A and F_B

($m=1200$ kg)

axis



$$\Sigma F = 0$$

$$F_A + F_B - mg = 0$$

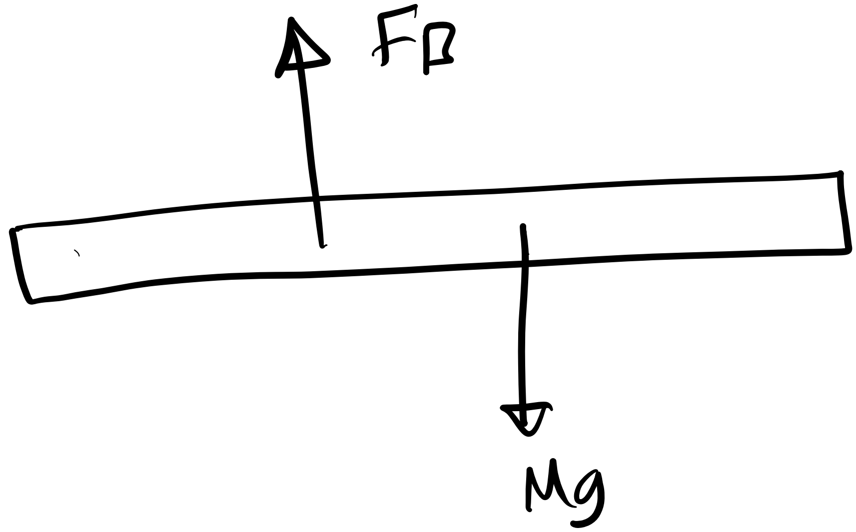
$$F_A + F_B = mg$$

$$\Sigma \tau = 0$$

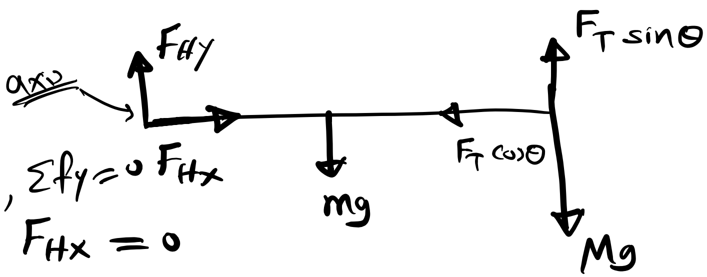
$$F_B(20) - mg(25) = 0$$

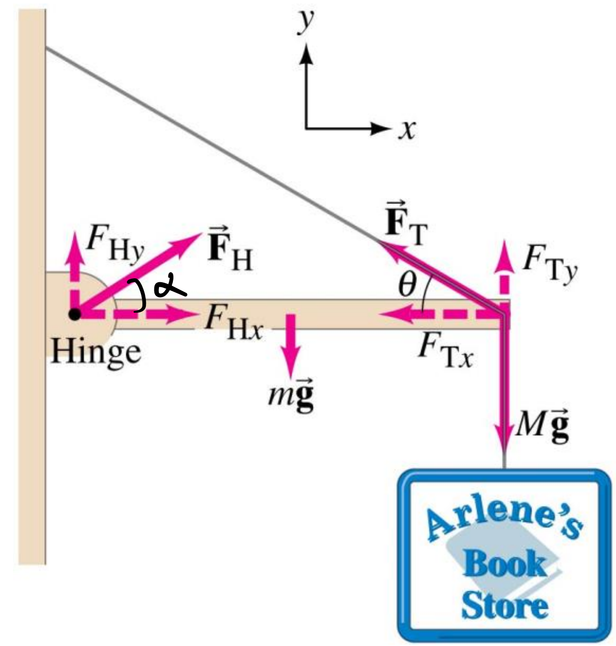
$$F_B = \frac{mg(25)}{20} = 14700\ N$$

$$F_A = mg - F_B$$
$$= 1200(9.8) - 14700 = -2940\ N$$


FB
Mg

- A uniform beam, 2.20 m long with mass 25 kg is mounted by a small hinge. If the supporting cable makes an angle 30°. Determine the components of the force that the (smooth) hinge exerts on the beam, and the tension in the cable.





28 kg

$\Sigma F_x = 0, \ \Sigma F_y = 0$

$F_T \cos\theta - F_{Hx} = 0$

$F_T \sin\theta + F_{Hy} - mg - Mg = 0$

$\Sigma \tau = 0$

$F_T \sin\theta (2.2) - Mg(2.2) - mg(1.1) = 0$

$F_T = \dfrac{(28)(9.8)(2.2) + 25(9.8)(1.1)}{(2.2)\sin 30^\circ}$

$F_T = 792 \text{ N}$

$F_{Hx} = F_T \cos\theta$
$= (792)\cos 30^\circ$
$= 686 \text{ N}$

$F_{Hy} = mg + Mg - F_T \sin\theta$

$F_{Hy} = 122 \text{ N}$

$F_H = \sqrt{686^2 + 122^2}$

$\alpha = \tan^{-1}\left(\dfrac{122}{686}\right)$

A 5.0-m-long ladder leans against a wall at a point 4.0 m above a cement floor. The ladder is uniform and has mass 9.0 kg Assuming the wall is frictionless, but the floor is not, determine the forces exerted on the ladder by the floor and by the wall.

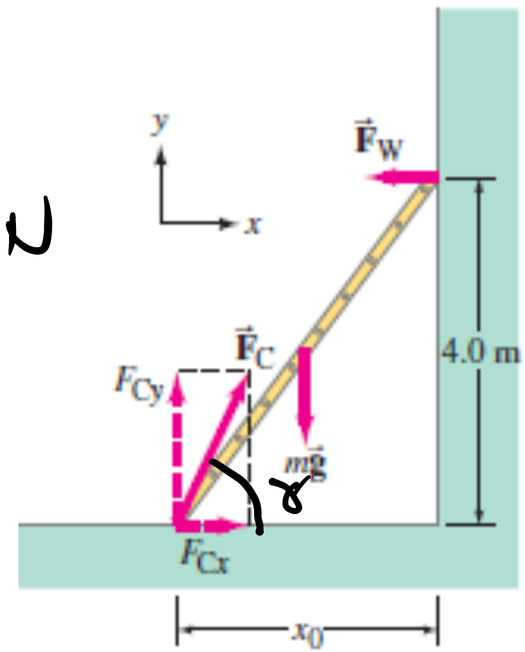
$\Sigma F = 0$

$F_{cx} - F_w = 0$

$F_{cy} - mg = 0 \Rightarrow F_{cy} = mg$

$= (9)(9.8)$

$= 88N$



$\Sigma \tau = 0$

$\sin\theta = \frac{4}{5}$

since $\alpha + \theta = 90°$

$\sin\alpha = \cos\theta$

$\cos\theta = \frac{3}{5}$

$F_w \sin\theta (5) - mg \sin\alpha (2.5) = 0$

$F_w \left(\frac{4}{\cancel{5}}\right)(\cancel{5}) = mg \left(\frac{3}{\cancel{5}_2}\right)(\cancel{2.5})$

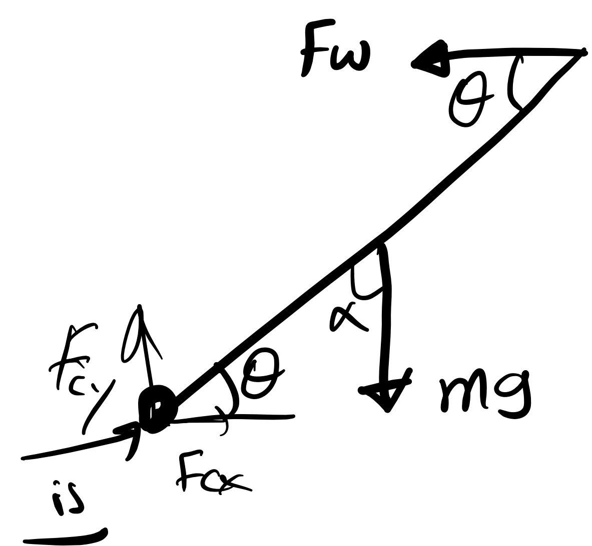
$4F_w = 1.5mg$

$F_w = 33N$

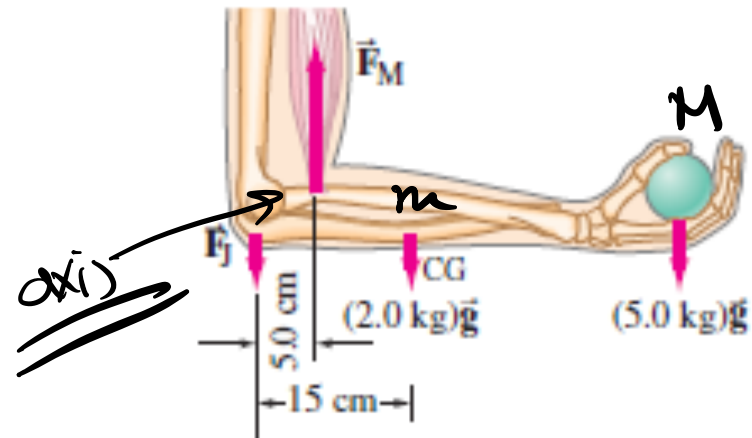
$F_{cx} = F_w = 33N$

$F_c = \sqrt{33^2 + 88^2}$

$\gamma = \tan^{-1}\left(\frac{88}{33}\right)$



- How much force must the biceps muscle exert when a 5.0-kg ball is held in the hand (*a*) with the arm horizontal and (*b*) when the arm is at a 45° angle as in Fig? The biceps muscle is connected to the forearm by a tendon attached 5.0 cm from the elbow joint. Assume that the mass of forearm and hand together is 2.0 kg and their CG is as shown.



(a)

$\Sigma F = 0$

$F_M - F_j - mg - Mg = 0$

$\Sigma \tau = 0$

$-Mg(0.35) - mg(0.15) + F_M(0.05) = 0$

$F_M = 400\ N$

$F_j = F_M - mg - Mg$

$= 400 - (7)(9.8)$

$\simeq 33$[illegible] N

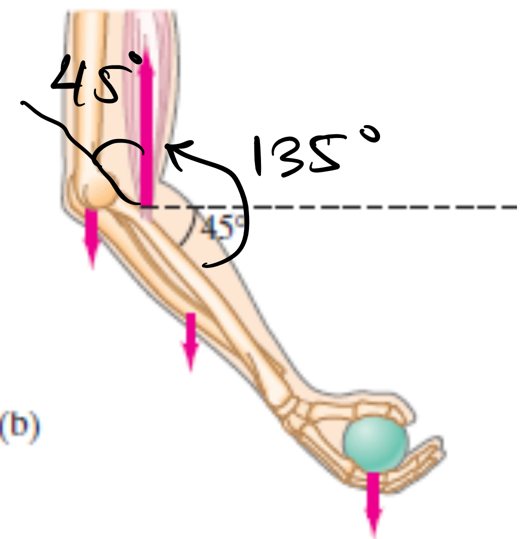
- How much force must the biceps muscle exert when a 5.0-kg ball is held in the hand (*a*) with the arm horizontal and (*b*) when the arm is at a 45° angle as in Fig? The biceps muscle is connected to the forearm by a tendon attached 5.0 cm from the elbow joint. Assume that the mass of forearm and hand together is 2.0 kg and their CG is as shown.

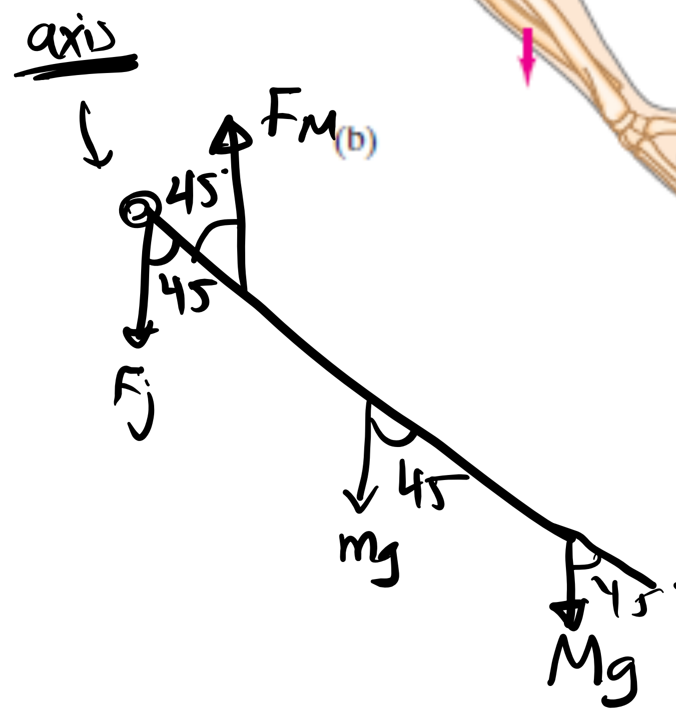
$$F_M - F_j - mg - Mg = 0$$

$$\Sigma\tau = 0$$

$$-Mg\sin\cancel{45^\circ}(0.35) - mg\sin\cancel{45}(0.15) + F_M\sin\cancel{45}(0.05) = 0$$

$$F_M = 400\,N$$





Calculate the magnitude and direction of the force acting on the fifth lumbar vertebra as represented in Fig. 9–14b.

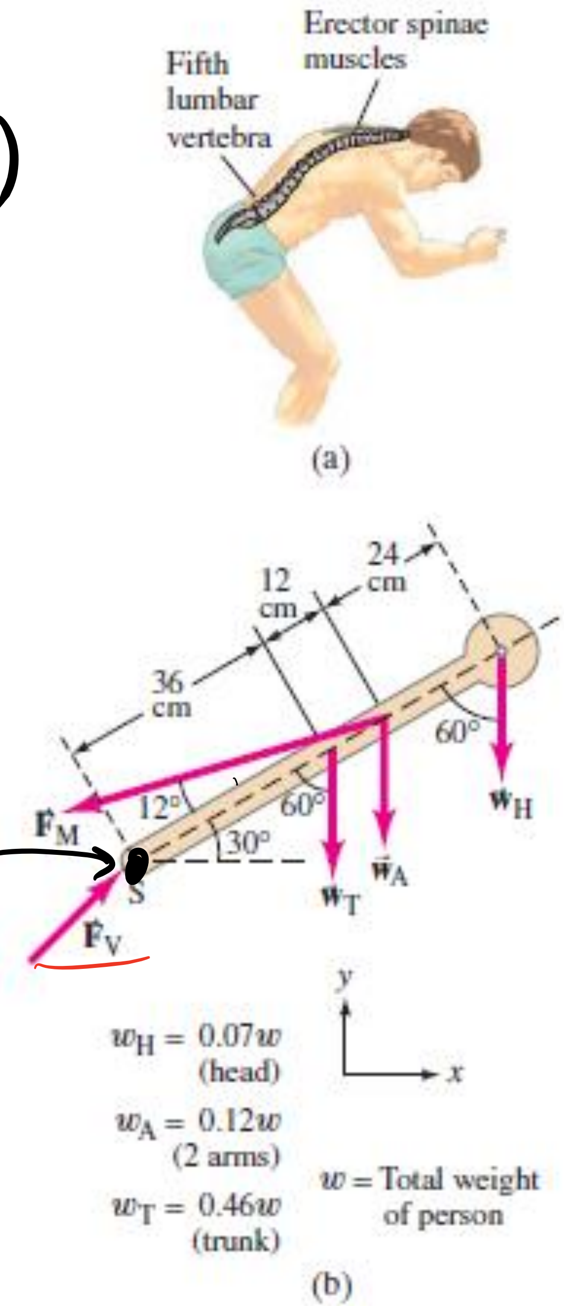
$$\sum \tau = 0$$

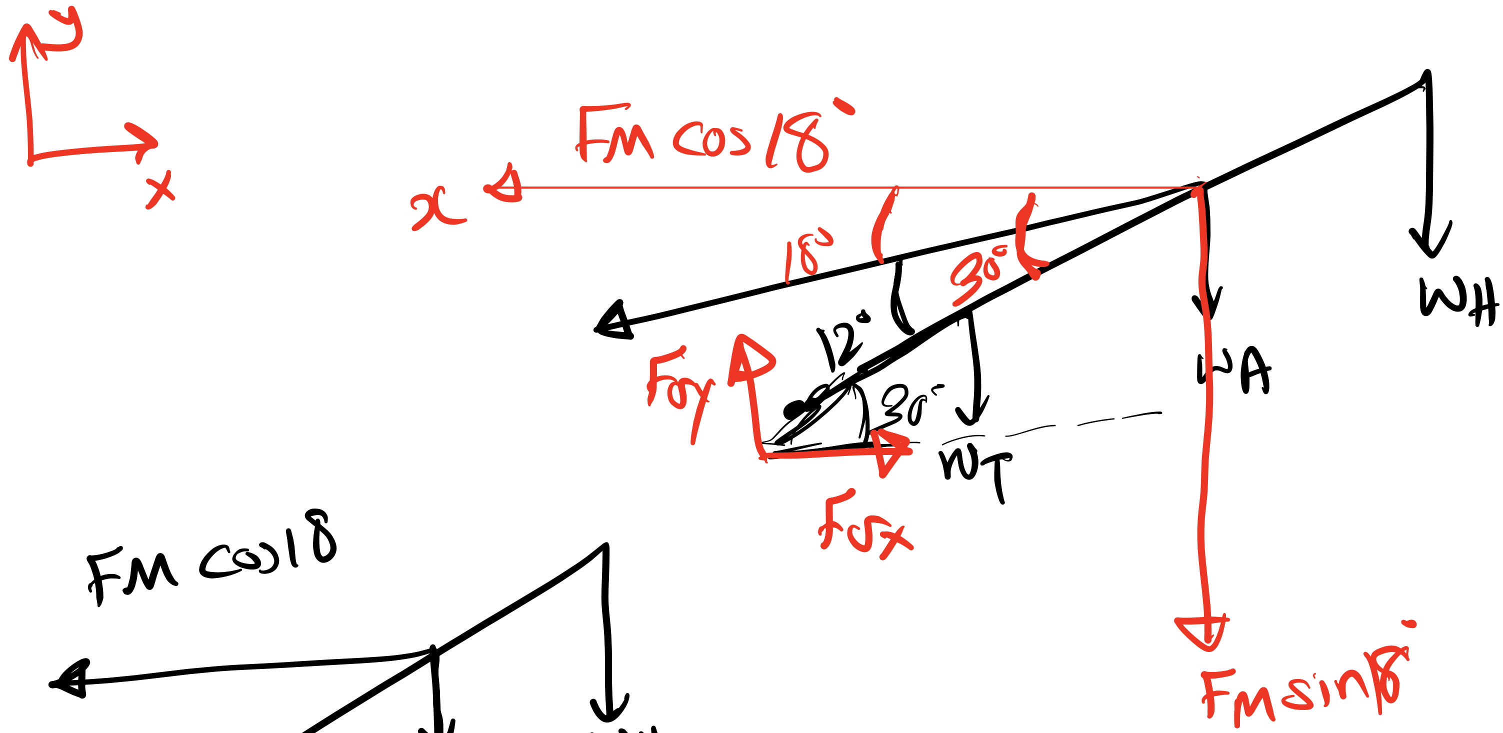
$$-w_H \sin 60\,(0.72) - w_A \sin 60\,(0.48) - w_T \sin 60\,(0.36) + F_M \sin 12^\circ\,(0.48) = 0$$

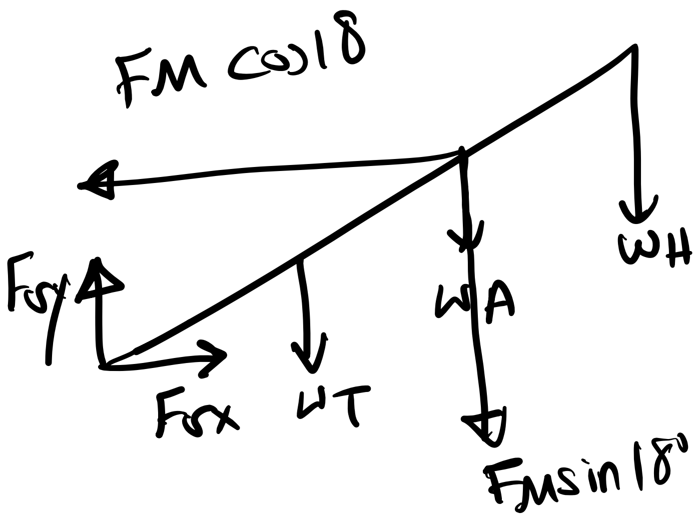
$$F_M = \frac{\left[(0.07w)(0.72)\sin 60 + (0.12w)(0.48)\sin 60 + (0.46w)(0.36)\sin 60\right]}{(0.48)\sin 12^\circ}$$

$$= 2.3\,W$$

axis







$$\sum F_x = 0$$

$$F_M \cos 18° - F_{rx} = 0 \Rightarrow F_{rx} = 2.25\,W$$

$$F_M \sin 18° + W_H + W_T + W_A - F_{ry} = 0$$

$$F_{ry} = 1.38\,W$$

$$F_r = \sqrt{F_{rx}^2 + F_{ry}^2}$$

$$= \sqrt{2.25^2 + 1.38^2} = 2.6\,W$$

$$\theta = \tan^{-1}\left(\frac{1.38}{2.25}\right) =$$

9-5 Elasticity; Stress and Strain

Hooke's law: the change in length is proportional to the applied force.

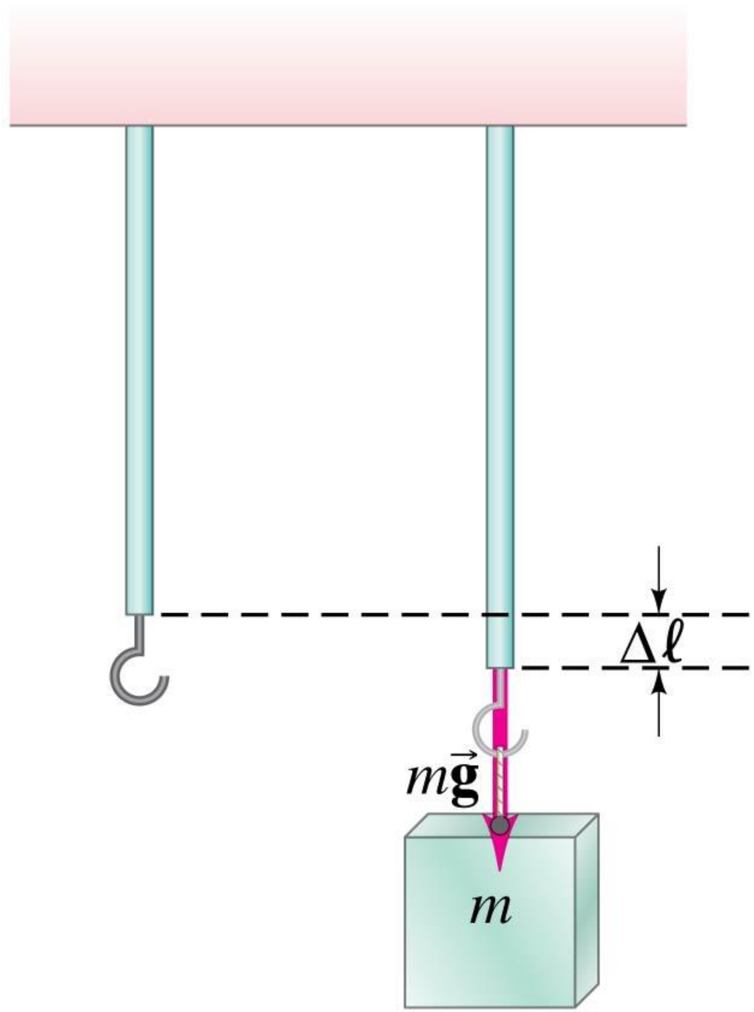
$F \propto \Delta l$

$$F = k\,\Delta\ell \qquad (9\text{-}3)$$

k = elastic constant

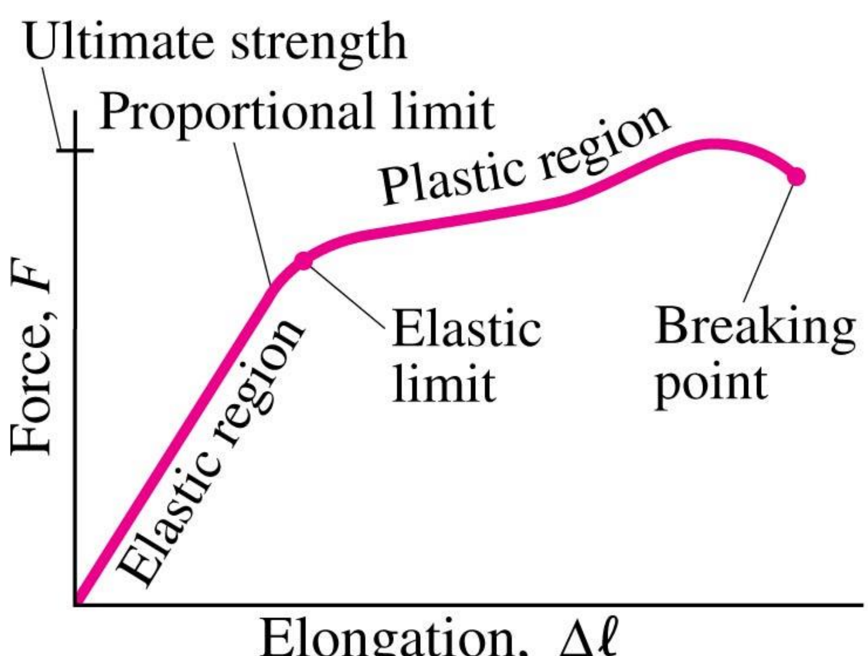
stress = F/A
unit N/m^2

strain = $\Delta l / l_0$
No unit



9-5 Elasticity; Stress and Strain

This proportionality holds until the force reaches the proportional limit. Beyond that, the object will still return to its original shape up to the elastic limit. Beyond the elastic limit, the material is permanently deformed, and it breaks at the breaking point.



9-5 Elasticity; Stress and Strain

The change in length of a stretched object depends not only on the applied force, but also on its length and cross-sectional area, and the material from which it is made.

The material factor is called Young's modulus, and it has been measured for many materials.

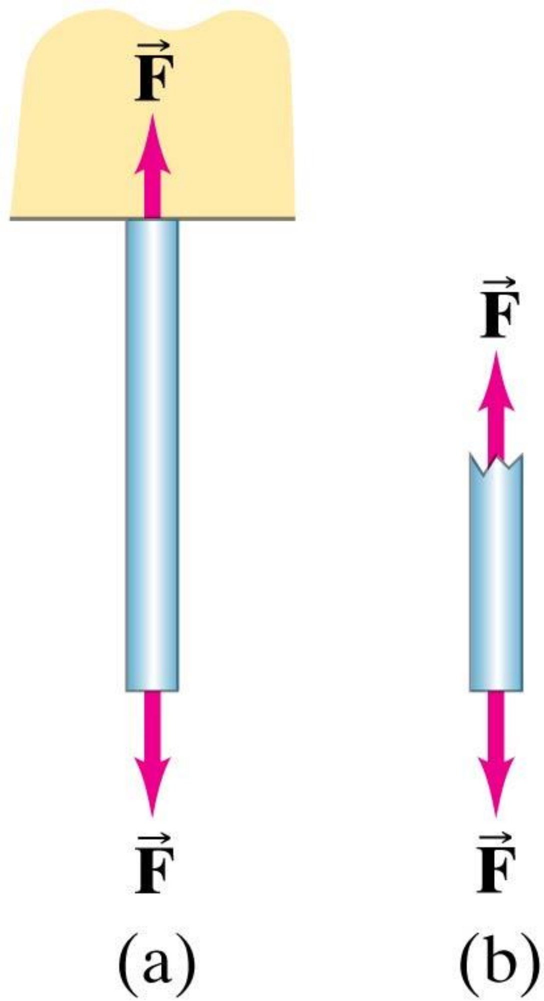
$$\text{stress} = \frac{\text{force}}{\text{area}} = \frac{F}{A}$$

The Young's modulus is then the stress divided by the strain.

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta \ell}{\ell_0}$$

9-5 Elasticity; Stress and Strain

In tensile stress, forces tend to stretch the object.



Young modulus

$Y = \dfrac{\text{stress}}{\text{strain}}$

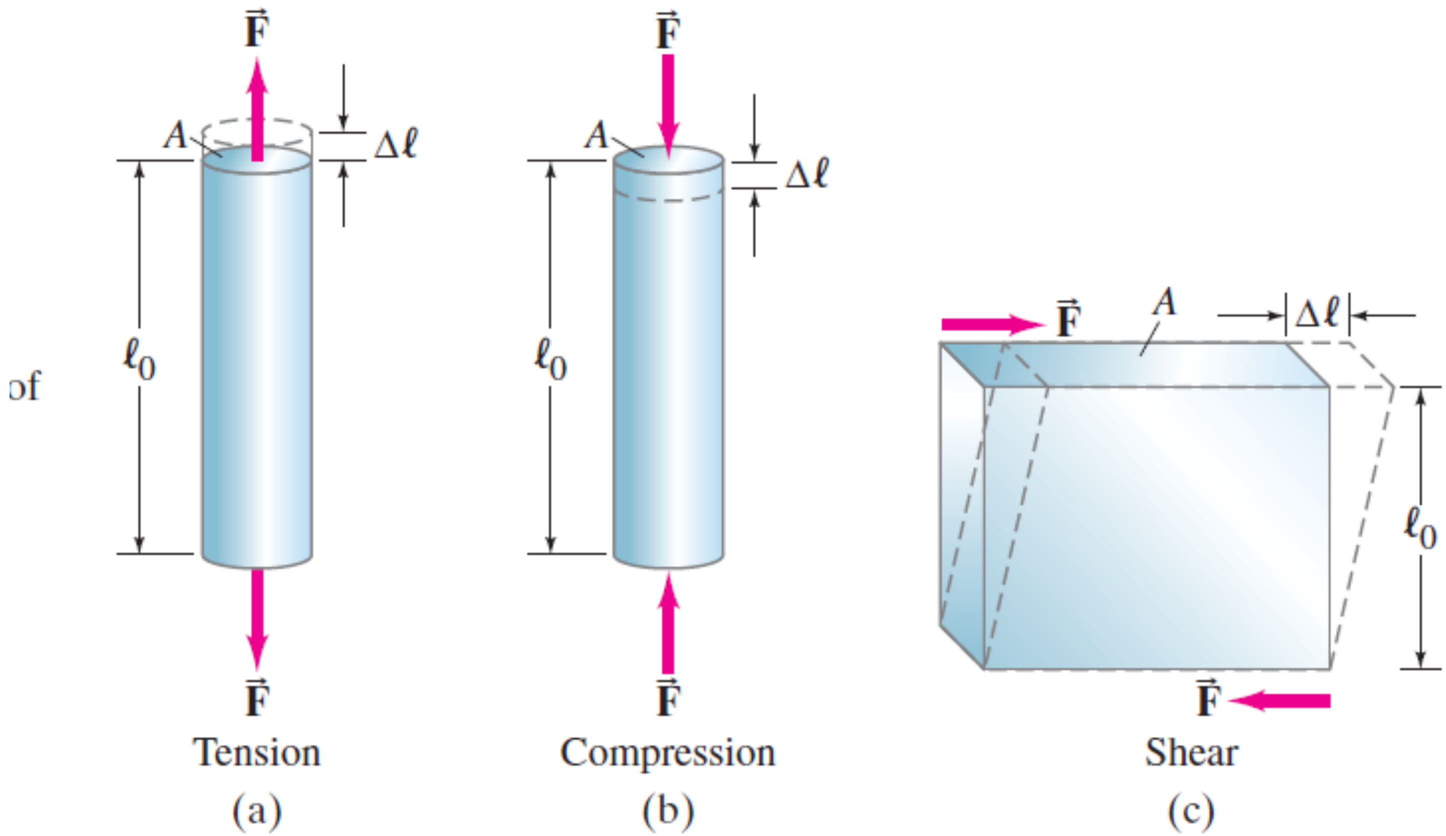
unit N/m^2

9-5 Elasticity; Stress and Strain

Compressional stress is exactly the opposite of tensional stress. These columns are under compression.

9-5 Elasticity; Stress and Strain

Shear stress tends to deform an object:



$$\Delta\ell = \frac{1}{G}\frac{F}{A}\ell_0$$

38. (I) A marble column of cross-sectional area 1.4 m^2 supports a mass of 25,000 kg. (*a*) What is the stress within the column? (*b*) What is the strain?

$$\text{stress} = \frac{F}{A} = \frac{25000 \times 9.8}{1.4} = 175000 \ N/m^2$$

$$\text{strain} = \frac{\Delta l}{l}$$

$$Y = \frac{\text{stress}}{\text{strain}} \Rightarrow 50 \times 10^9 = \frac{175000}{\text{strain}} \Rightarrow \text{strain} = \frac{175000}{50 \times 10^9}$$

from the table

39. (I) By how much is the column in Problem 38 shortened if it is 8.6 m high?

$$\text{strain} = \frac{\Delta l}{l}$$

$$\frac{17500}{50 \times 10^9} = \frac{\Delta l}{8.6} \Rightarrow$$

43. (II) A 15-cm-long tendon was found to stretch 3.7 mm by a force of 13.4 N. The tendon was approximately round with an average diameter of 8.5 mm. Calculate Young's modulus of this tendon.

$L_0 = 15\text{ cm}$

$\Delta l = 3.7\text{ mm}$

$D = 8.5\text{ mm}$

$F = 13.4\text{ N}$

$$\text{strain} = \frac{3.7}{150}$$

$$\boxed{\text{stress} = \frac{13.4}{\left(\frac{8.5}{2} \times 10^{-3}\right)^2 \pi}}$$

$$Y = \frac{\text{stress}}{\text{strain}} = 9.6 \times 10^{6}\ \text{N/m}^2$$

Fracture

If the stress on a solid object is too great, the object fractures, or breaks

TABLE 9–2 Ultimate Strengths of Materials (force/area)

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)	Shear Strength (N/m^2)
Iron, cast	170×10^6	550×10^6	170×10^6
Steel	$500–2500 \times 10^6$	500×10^6	250×10^6
Brass	250×10^6	250×10^6	200×10^6
Aluminum	200×10^6	200×10^6	200×10^6
Concrete	2×10^6	20×10^6	2×10^6
Brick		35×10^6	
Marble		80×10^6	
Granite		170×10^6	
Wood (pine) (parallel to grain)	40×10^6	35×10^6	5×10^6
(perpendicular to grain)		10×10^6	
Nylon	500×10^6		
Bone (limb)	130×10^6	170×10^6	

The steel piano wire was 1.60 m long with a diameter of 0.20 cm. Approximately what tension force would break it?

ultimate strength $= 2500 \times 10^{6}$ N/m^2

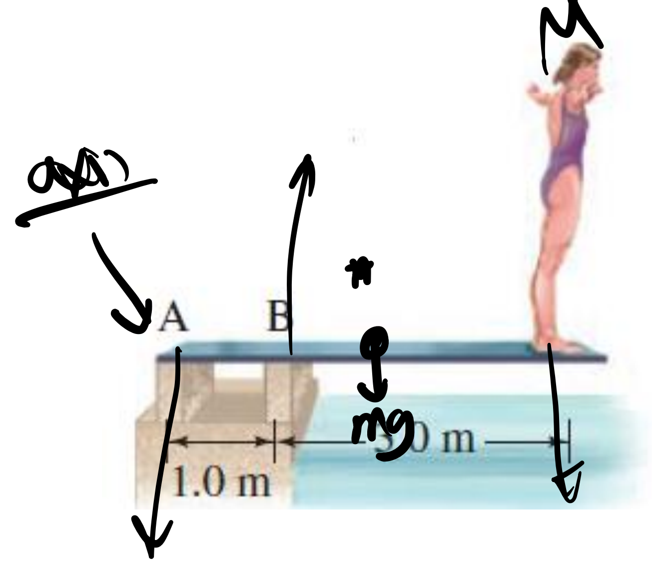
stress

$$= \frac{F}{A}$$

$$2500 \times 10^{6} = \frac{F}{\pi (0.1 \times 10^{-2})^{2}}$$

$$F = 2500 \times 10^{6} \times \pi \times 1 \times 10^{-6}$$

5. (II) Calculate the forces F_A and F_B that the supports exert on the diving board of Fig. 9–49 when a 52-kg person stands at its tip. (*a*) Ignore the weight of the board. (*b*) Take into account the board's mass of 28 kg. Assume the board's CG is at its center.



ⓐ $F_B - Mg - F_A = 0$

$F_B(1) - 4Mg = 0$

$F_B = 4mg$

$F_A = F_B - Mg$

$F_A = 3Mg$

ⓑ $F_B - Mg - mg - F_A = 0$

$F_B(1) - 4Mg - 2mg = 0$

$F_B = 4Mg + 2mg$

$= 2587\,N$

$F_A = 2587 - 52(9.8) - 28(9.8)$

$= 1803\,N$

16. (II) Calculate F_A and F_B for the beam shown in Fig. 9–56. The downward forces represent the weights of machinery on the beam. Assume the beam is uniform and has a mass of 280 kg.

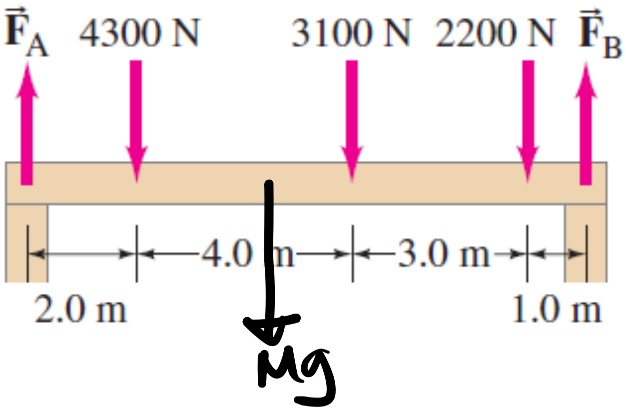


FIGURE 9–56
Problem 16.

$$F_A + F_B - 4300 - 3100 - 2200 - 280(9.8) = 0$$

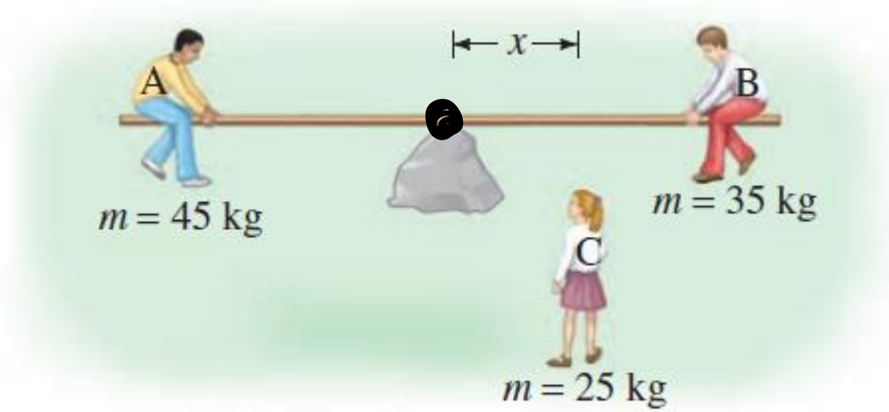
$$F_A + F_B = 12344\ N$$

$$F_B(10) - 2200(9) - 3100(6) - 280(5)(9.8) - 4300(2) = 0$$

$$F_B = 6072\ N$$

$$F_A = 6272\ N$$

17. (II) Three children are trying to balance on a seesaw, which includes a fulcrum rock acting as a pivot at the center, and a very light board 3.2 m long (Fig. 9–57). Two playmates are already on either end. Boy A has a mass of 45 kg, and boy B a mass of 35 kg. Where should girl C, whose mass is 25 kg, place herself so as to balance the seesaw?



$$m_A \cancel{g}(1.6) - m_B \cancel{g}(1.6) - m_C \cancel{g} x = 0$$

$$(45)(1.6) - 35(1.6) - 25x = 0$$

$$16 = 25x$$

$$x = \frac{16}{25} = 0.64 \text{ m}$$

18. (II) A shop sign weighing 215 N hangs from the end of a uniform 155-N beam as shown in Fig. 9–58. Find the tension in the supporting wire (at 35.0°), and the horizontal and vertical forces exerted by the hinge on the beam at the wall. [*Hint*: First draw a free-body diagram.]

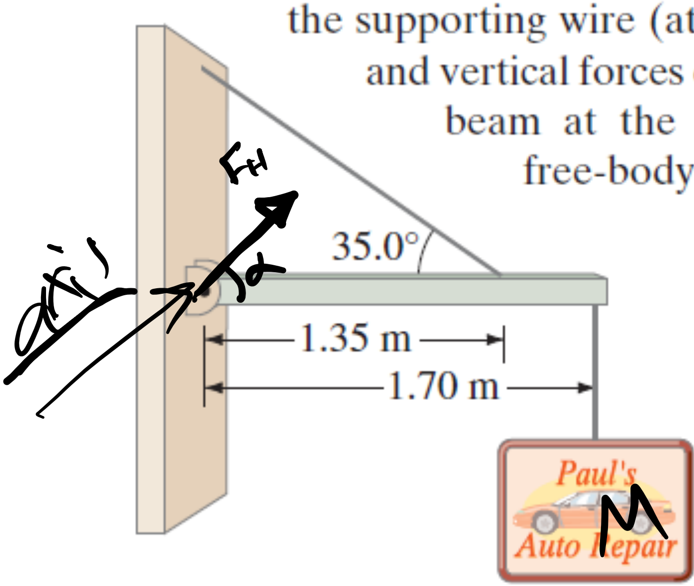
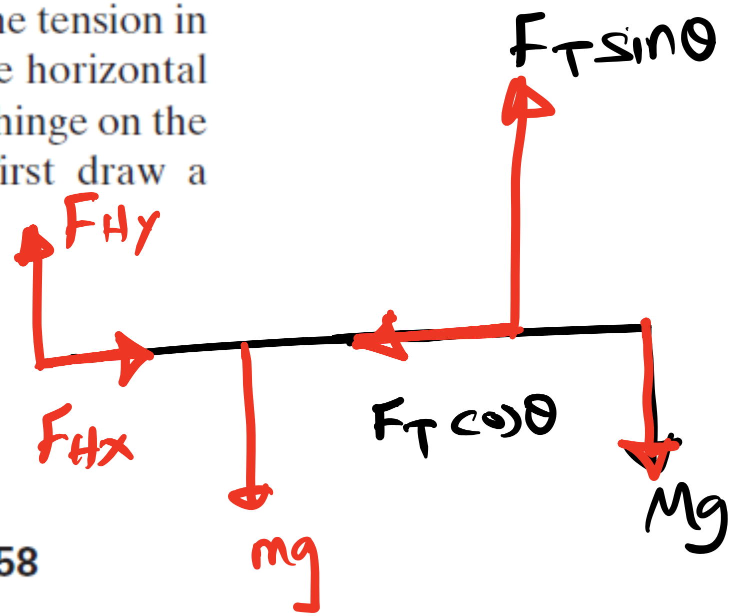


FIGURE 9–58
Problem 18.



$$F_T \sin 35 + F_{Hy} - Mg - mg = 0$$

$$F_{Hx} - F_T \cos 35 = 0$$

$$F_T \sin 35 (1.35) - Mg(1.7) - mg(0.85) = 0$$

$$F_T \sin 35 (1.35) - 215(1.7) - 155(0.85) = 0$$

$$F_T = 642 \text{ N}$$

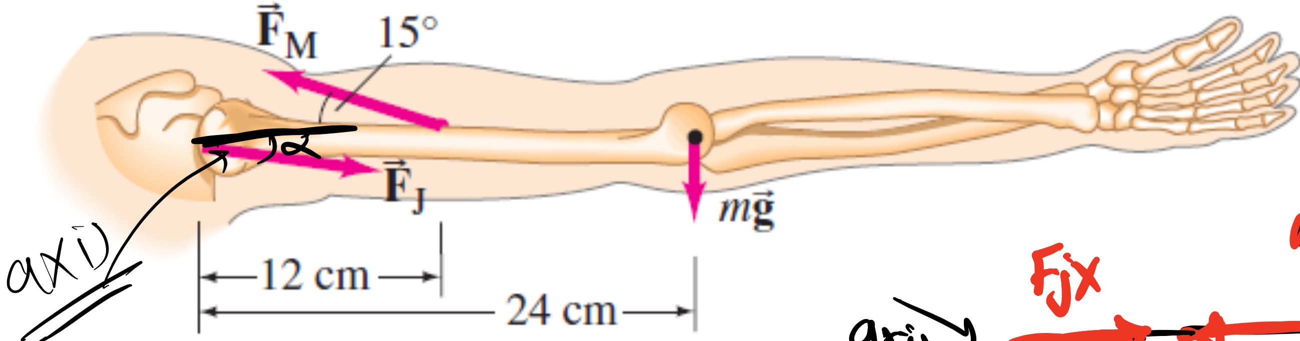
$$F_{Hx} = 642 \cos 35^\circ = 526 \text{ N}$$

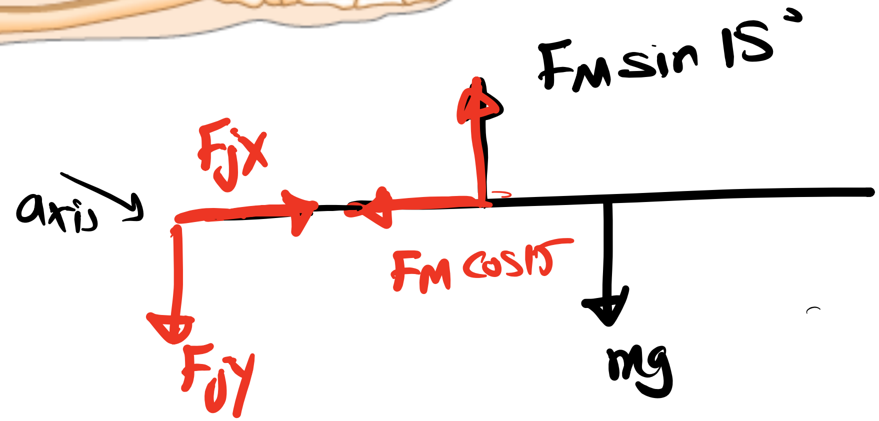
$$F_{Hy} = Mg + mg - F_T \sin 35 = 215 + 155 - 642 \sin 35 = 1.76 \text{ N}$$

$$F_H = \sqrt{526^2 + 1.76^2} \simeq 527 \text{ N} \checkmark$$

$$\alpha = \tan^{-1}\left(\frac{1.76}{526}\right)$$

32. (II) (*a*) Calculate the magnitude of the force, F_M, required of the "deltoid" muscle to hold up the outstretched arm shown in Fig. 9–71. The total mass of the arm is 3.3 kg. (*b*) Calculate the magnitude of the force F_J exerted by the shoulder joint on the upper arm and the angle (to the horizontal) at which it acts.





$$F_M \sin 15 - F_{jy} - mg = 0$$

$$F_M \cos 15 - F_{jx} = 0$$

$$F_M \sin 15° (0.12) - mg\,(0.24) = 0$$

$$F_M = \frac{(3.3)(9.8)(0.24)}{(0.12)\sin 15} = 250\,N$$

$$F_{jy} = F_M \sin 15 - mg$$
$$= 32.4$$

$$F_{jx} = 250 \cos 15$$
$$= 241.5\,N$$

$$F_j = \sqrt{241.5^2 + 32.4^2} = 243\,N \qquad \alpha = \tan^{-1}\left(\frac{32.4}{241.5}\right) = 7.6°$$

46. (I) The femur bone in the human leg has a minimum effective cross section of about 3.0 cm^2 ($= 3.0 \times 10^{-4}\ m^2$). How much compressive force can it withstand before breaking?

$A = 3 \times 10^{-4}$

ultimate strength $= 170 \times 10^{6}$

$\text{Stress} = \dfrac{F}{A}$

$170 \times 10^{6} = \dfrac{F}{3 \times 10^{-4}} \Rightarrow F = 510 \times 10^{2}$

$= 5.1 \times 10^{4}\ N$

50. (II) An iron bolt is used to connect two iron plates together. The bolt must withstand shear forces up to about 3300 N. Calculate the minimum diameter for the bolt, based on a safety factor of 7.0.

ultimate $= 170 \times 10^6$

max stress $= \dfrac{170 \times 10^6}{7} = \dfrac{3300}{A}$

$$A = \frac{3300 \times 7}{170 \times 10^6}$$