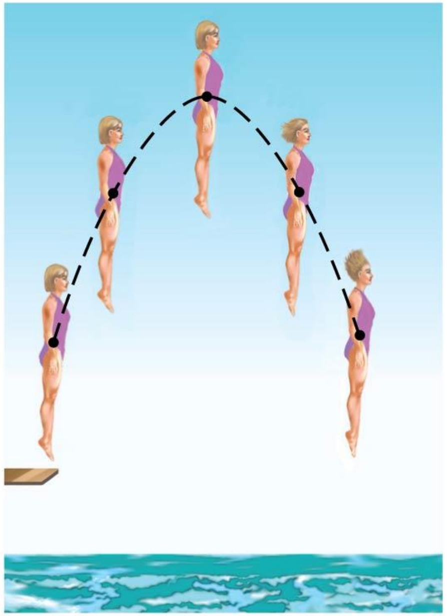
Chapter 7
Linear Momentum
(center of mass only)

Center of Mass

In (a), the diver's motion is pure translation; in (b) it is translation plus rotation.

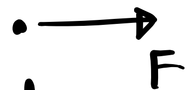
There is one point that moves in the same path a particle would take if subjected to the same force as the diver. This point is called the center of mass (CM).



(a)

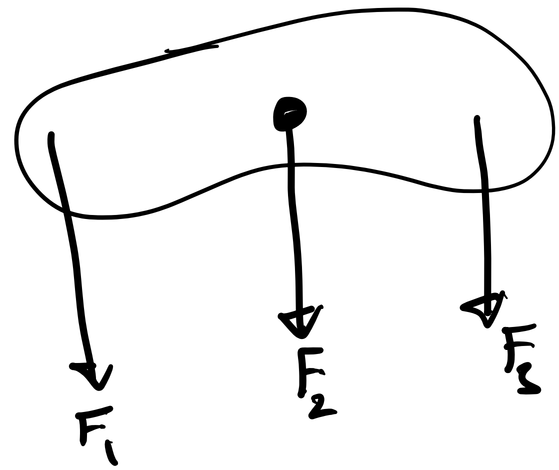
(b)

Force acting on a particle



only translation motion

rigid body



$F_1 = F_2 = F_3$

each one has diffrent consequences

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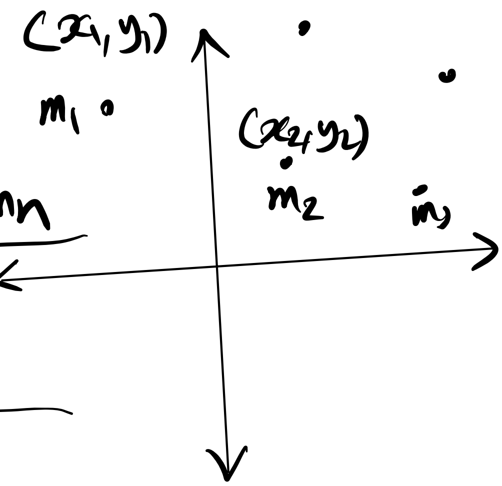
to find the coordinates of the CM



$$X_{cm} = \frac{x_1(m_1) + x_2(m_2) + \cdots + x_n m_n}{m_1 + m_2 + \cdots + m_n}$$

$$X_{cm} = \frac{\sum m_i X_i}{\sum m_i}$$

$$Y_{cm} = \frac{\sum m_i y_i}{}$$

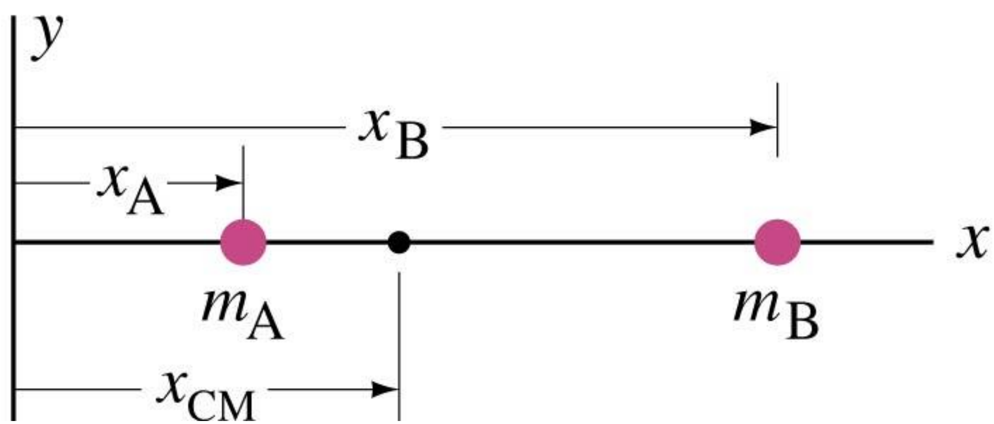
$\sum m_i$

For two particles, the center of mass lies closer to the one with the most mass:

$$x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M},$$

where M is the total mass.

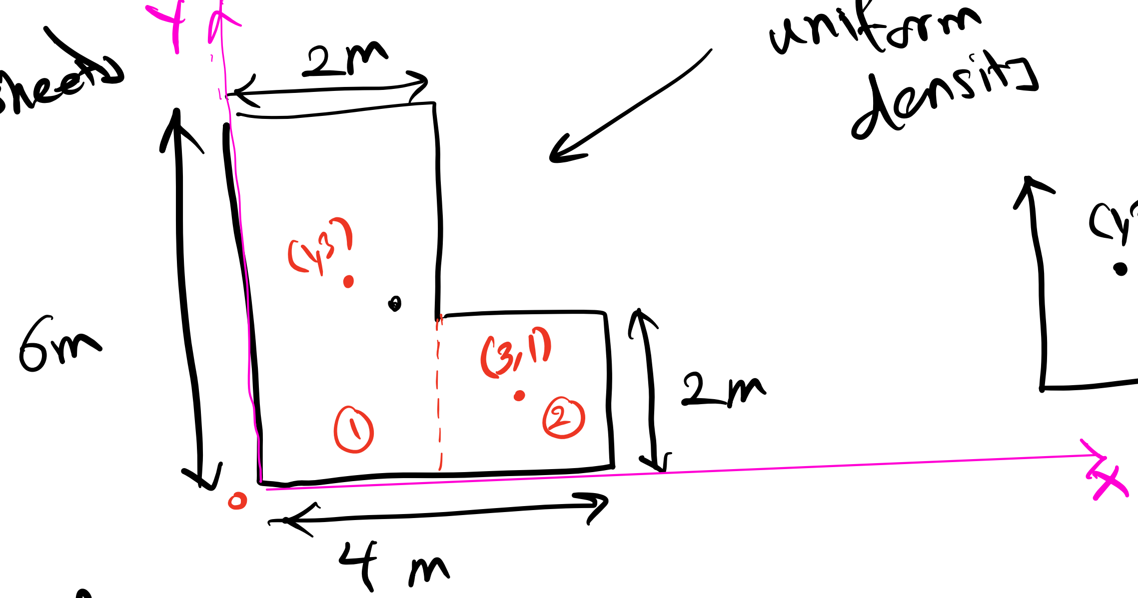
$m_A = 6$ kg
$m_B = 4$ kg
$x_A = 10$ cm
$x_B = 40$ cm

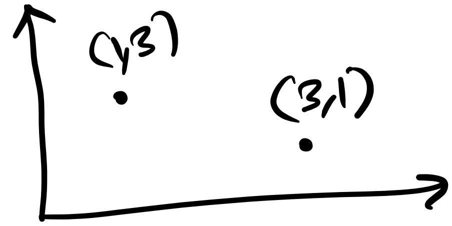


$$x_{cm} = \frac{6 \times 10 + 4 \times 40}{6+4} = 22 \text{ cm}$$

sheets — uniform density

Find CM?





assume density is d

$m_1 = d \times \text{area}$
$= d \times 12 = 12d$

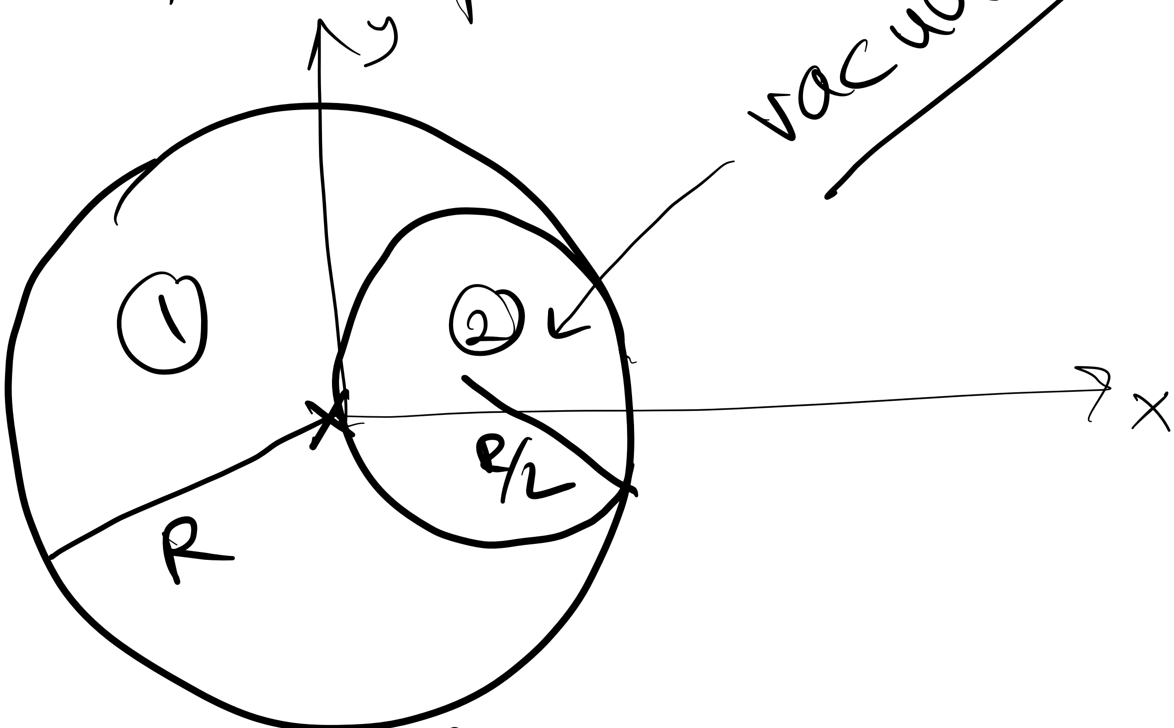
$m_2 = d \times \text{area} = 4d$
$= d \times 4$

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{12d(1) + 4d(3)}{16d} = \frac{24d}{16d} = 1.5$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2}{m_1 + m_2} = \frac{12(d)(3) + 4d(1)}{16d} = \frac{40d}{16d} = 2.5$$

$CM = (1.5, 2.5)$

Calculate the CM of the sphere



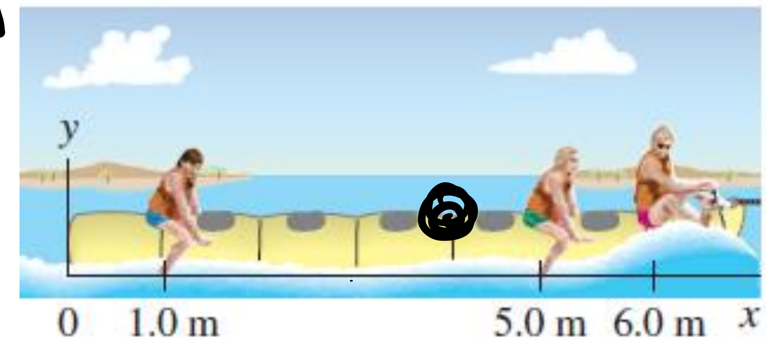
$$m_1 = \frac{4}{3}\pi R^3 d \Rightarrow CM = 0$$

$$m_2 = -\frac{4}{3}\pi \left(\frac{R}{2}\right)^3 d \Rightarrow CM = \frac{R}{2}$$

$$x_{cm} = \frac{\frac{4}{3}\pi R^3 d\,(0) + -\frac{4}{3}\pi \left(\frac{R}{2}\right)^3 d \frac{R}{2}}{\frac{4}{3}\pi R^3 d - \frac{4}{3}\pi \left(\frac{R}{2}\right)^3 d}$$

$$= \frac{-\frac{1}{8}\frac{R}{2}}{1-\frac{1}{8}} = \frac{-\frac{1}{8}\frac{R}{2}}{\frac{7}{8}} \quad R$$

EXAMPLE 7–12 **CM of three guys on a raft.** On a lightweight (air-filled) "banana boat," three people of roughly equal mass m sit along the x axis at positions $x_A = 1.0\text{ m}$, $x_B = 5.0\text{ m}$, and $x_C = 6.0\text{ m}$, measured from the left-hand end as shown in Fig. 7–23. Find the position of the CM. Ignore the mass of the boat.



$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

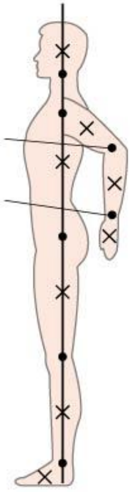
$$= \frac{m(1) + m(5) + m(6)}{3m}$$

$$= \frac{12}{3} = 4\text{ m}$$

7-9 CM for the Human Body

The x's in the small diagram mark the CM of the listed body segments.

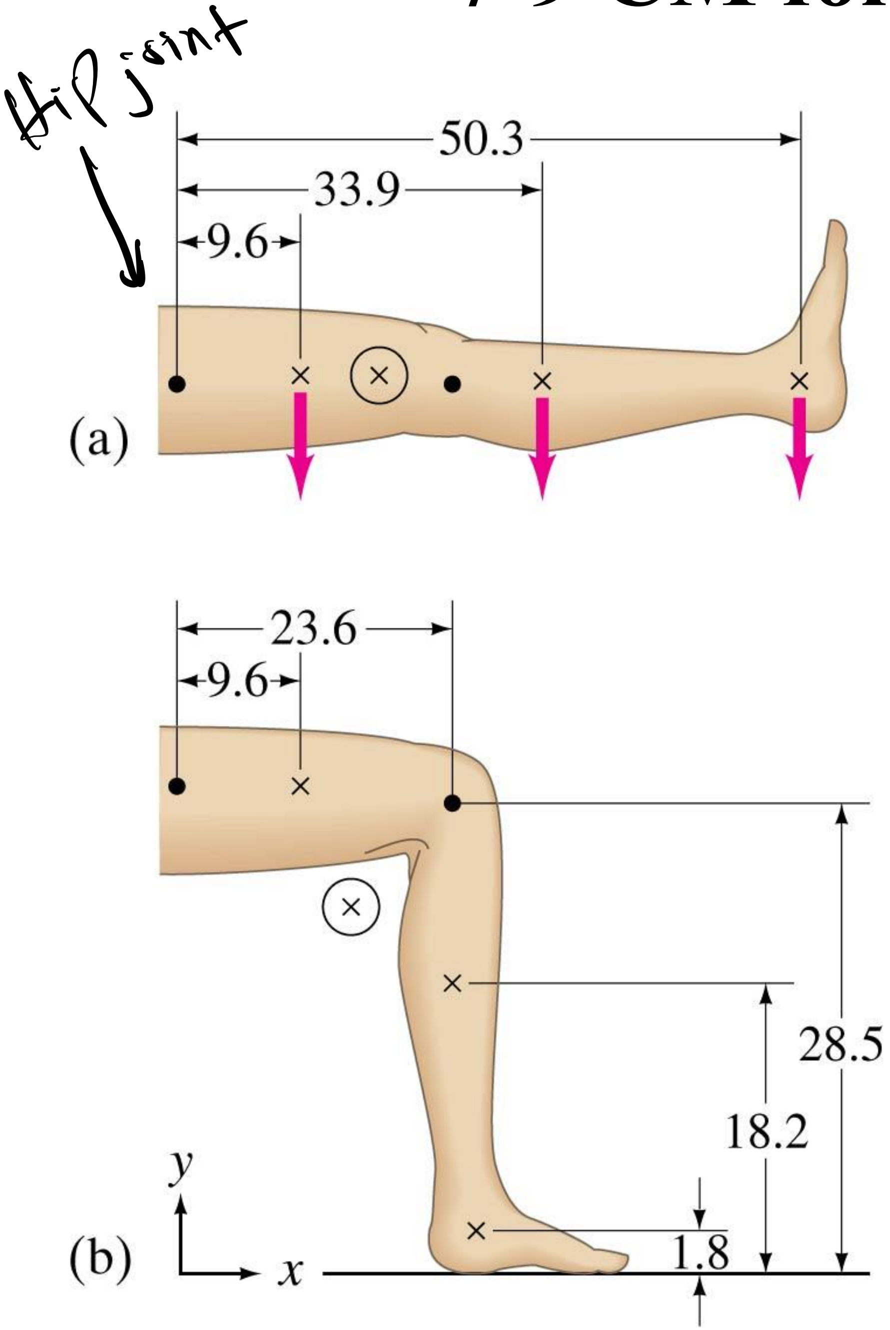
TABLE 7–1 Center of Mass of Parts of Typical Human Body, given as %
(full height and mass = 100 units)



Distance of Hinge Points from Floor (%)	Hinge Points (•) (Joints)	Center of Mass (×) (% Height Above Floor)		Percent Mass
		Head	93.5%	6.9%
91.2%	Base of skull on spine	Trunk and neck	71.1%	46.1%
81.2%	Shoulder joint	Upper arms	71.7%	6.6%
62.2%‡	elbow			
		Lower arms	55.3%	4.2%
46.2%‡	wrist			
52.1%	Hip joint	Hands	43.1%	1.7%
		Upper legs (thighs)	42.5%	21.5%
28.5%	Knee joint			
		Lower legs	18.2%	9.6%
4.0%	Ankle joint	Feet	1.8%	3.4%
		Body CM =	58.0%	100.0%

‡ For arm hanging vertically.

7-9 CM for the Human Body

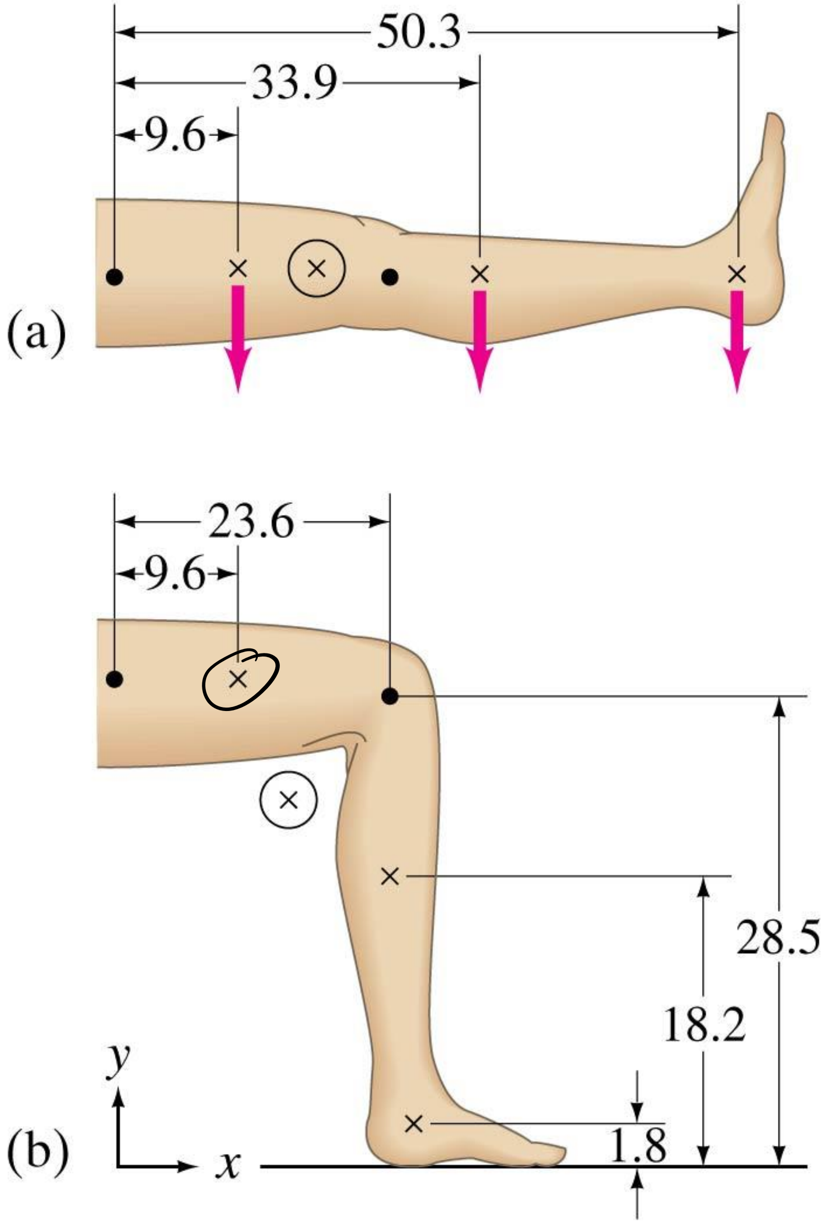


The location of the center of mass of the leg (circled) will depend on the position of the leg.

52.1% − 28.5

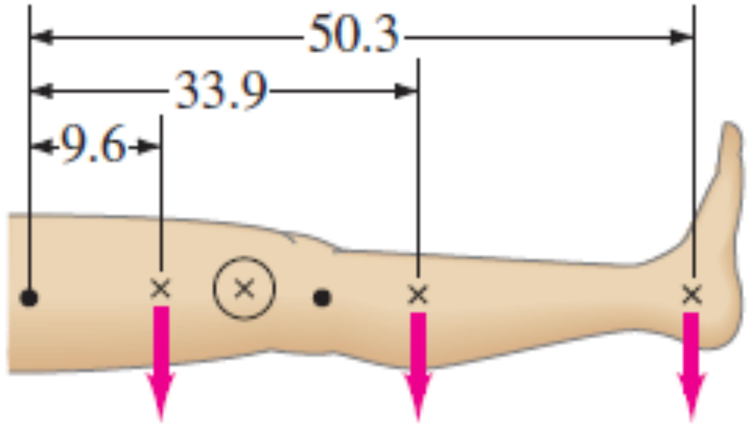
= 23.6

7-9 CM for the Human Body



The location of the center of mass of the leg (circled) will depend on the position of the leg.

CM for the Human Body



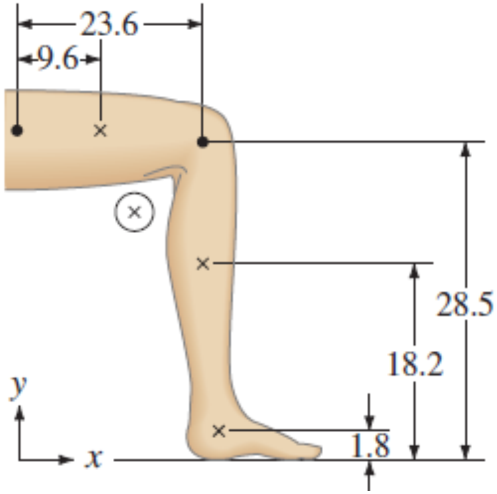
$52.1\% - 42.5\% = 9.6\%$

$52.1\% - 18.2\% = 33.9\%$

$52.1\% - 1.8\% = 50.3\%$

$$x_{cm} = \frac{(21.5)(9.6) + (9.6)(33.9) + (3.4)(50.3)}{21.5 + 9.6 + 3.4}$$

(b)



$$x_{cm} = \frac{(21.5)(9.6) + (9.6)(23.6) + (3.4)(23.6)}{(21.5) + (9.6) + (3.4)}$$

$$y_{cm} = \frac{(3.4)(1.8) + (9.6)(18.2) + (21.5)(28.5)}{(21.5) + (9.6) + 3.4}$$

48. (II) Three cubes, of side ℓ_0, $2\ell_0$, and $3\ell_0$, are placed next to one another (in contact) with their centers along a straight line as shown in Fig. 7–38. What is the position, along this line, of the CM of this system? Assume the cubes are made of the same uniform material.

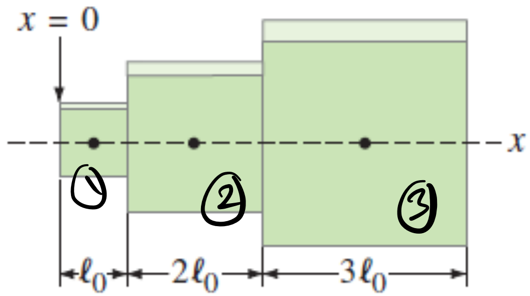


FIGURE 7–38
Problem 48.

density = d

$CM_1 = \frac{\ell_0}{2}$

$CM_2 = 2\ell_0$

$CM_3 = 4.5\ell_0$

$m_1 = d\,\ell_0^3$

$m_2 = d\,8\,\ell_0^3$

$m_3 = 27\,d\,\ell_0^3$

$m_2 = 8\,m_1$

$m_3 = 27\,m_1$

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

$$= \frac{m_1\left(\frac{\ell_0}{2}\right) + 8m_1(2\ell_0) + 27m_1(4.5\ell_0)}{36\,m_1}$$

$$= 3.8\,\ell_0$$

50. (III) Determine the CM of the uniform thin L-shaped construction brace shown in Fig. 7–40.

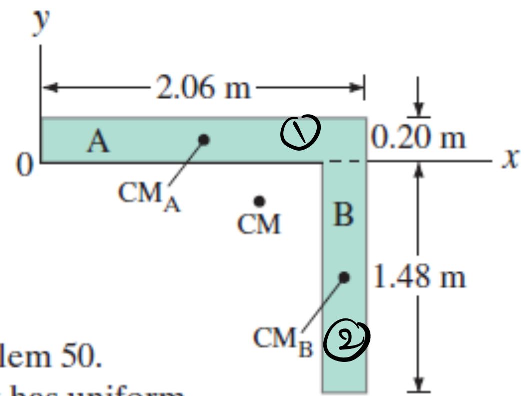


FIGURE 7–40 Problem 50. This L-shaped object has uniform thickness d (not shown).

$CM_1 = (1.03, 0.1)$

$CM_2 = (1.96, \bar{0}.74)$

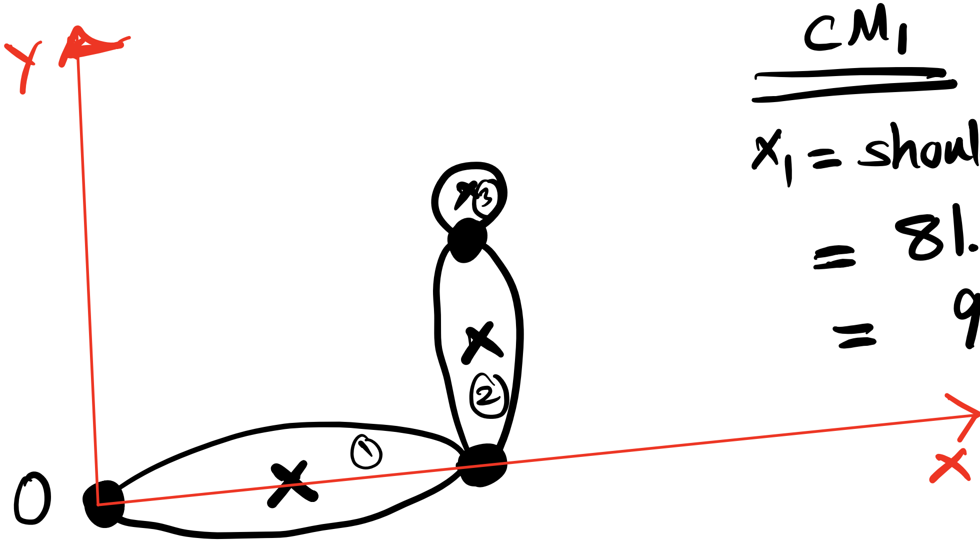
$m_1 = d \times (\text{area})$
$= d \times (2.06 \times 0.2)$
$= 0.412 d$

$m_2 = d \times (1.48 \times 0.2)$
$= 0.296 d$

$$x_{cm} = \frac{0.412 d (1.03) + 0.296 d (1.96)}{0.412 d + 0.296 d} = 1.4$$

$$y_{cm} = \frac{0.412 d (0.1) + 0.296 d (-0.74)}{0.412 d + 0.296 d} = -0.25$$

*53. (II) Use Table 7–1 to calculate the position of the CM of an arm bent at a right angle. Assume that the person is 155 cm tall.



CM_1

x_1 = shoulder joint − CM upperarm

$= 81.2 - 71.7$

$= 9.5$

$y_1 = 0$

CM_2

x_2 = shoulder joint − elbow joint

$= 81.2 - 62.2$

$= 19.0$

y_2 = elbow joint − CM of lower arm

$= 62.2 - 55.3$

$= 6.9$

CM_3

$X_3 = X_2 = 19$

$y_3 =$ elbow − cm of hand

$= 62.2 - 43.1$

$= 19.1$

$m_1 = 6.6$ $m_2 = 4.2$ $m_3 = 1.7$

$$X_{cm} = \frac{(6.6)(9.5) + (4.2)(19) + 1.7 \times 19}{6.6 + 4.2 + 1.7} = 14\%$$

$$Y_{cm} = \frac{(6.6)(0) + (4.2)(6.9) + (1.7)(19.1)}{6.6 + 4.2 + 1.7}$$

$= 4.9\%$

$X_{cm} = 155 \times \frac{14}{100} =$

$Y_{cm} = 155 \times \frac{4.9}{100} =$