Chapter 6

Work and Energy

© 2016 Pearson Education, Ltd.

Work Done by a Constant Force

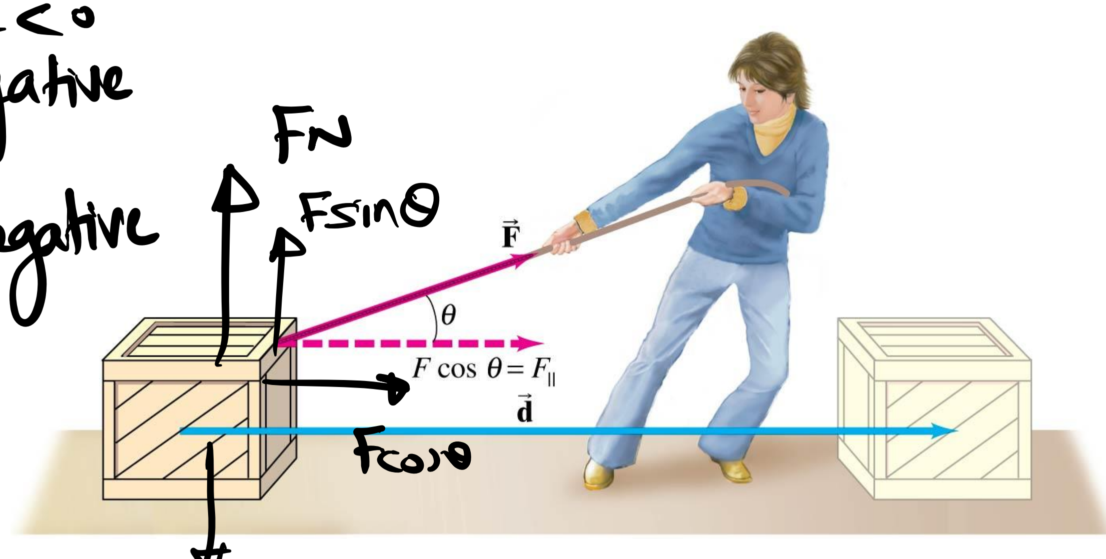
The work done by a constant force is defined as the distance moved multiplied by the component of the force in the direction of displacement: $W = Fd\cos\theta$

In the SI system, the units of work are joules:

$1\ \text{J} = 1\ \text{N} \cdot \text{m}$

Work is a scalar quantity—it has no direction, but only magnitude, which can be positive or negative or zero

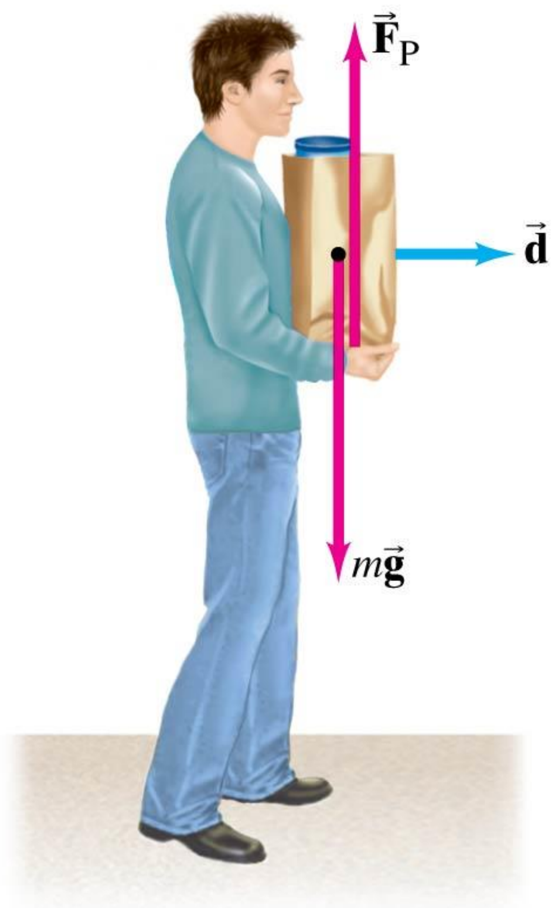
$0 \le \theta < 90$	$\theta = 90^\circ$	$90 < \theta \le 180$
$\cos\theta > 0$ positive $\Downarrow$ W positive	$W = 0$	$\cos\theta < 0$ negative $\Downarrow$ W negative



© 2016 Pearson Education, Ltd.

F_g

A force can be exerted on an object and yet do no work.

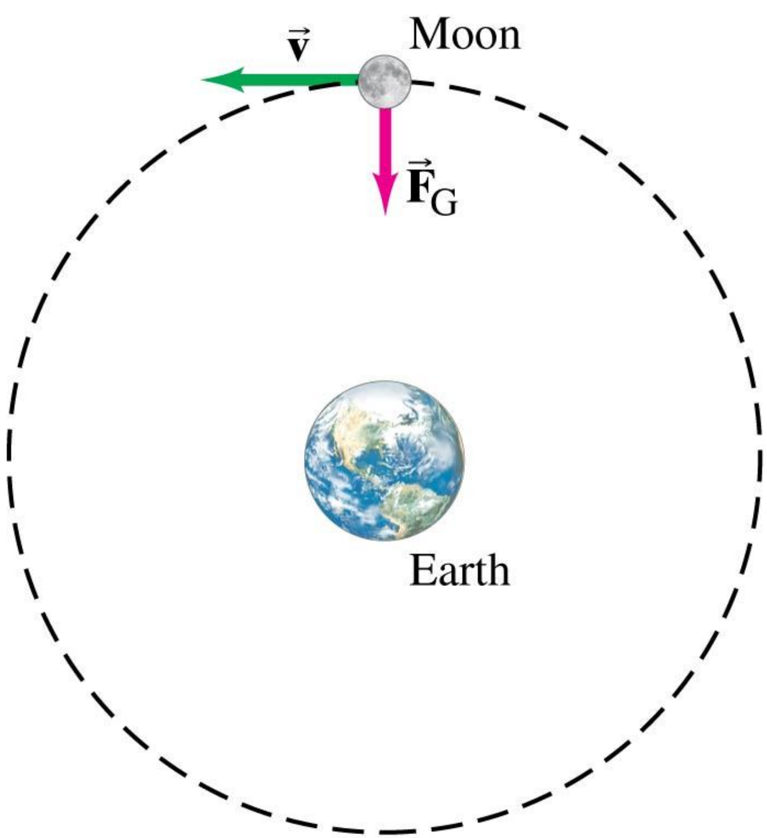


The force he exerts has no component in the direction of motion.

when a particular force is perpendicular to the displacement, no work is done by that force.

© 2016 Pearson Education, Ltd.

A force can be exerted on an object and yet do no work.



Centripetal forces do no work, as they are always perpendicular to the direction of motion.

© 2016 Pearson Education, Ltd.

What is work and how it is related to energy?

Work : amount of energy transfer caused by exerting force.

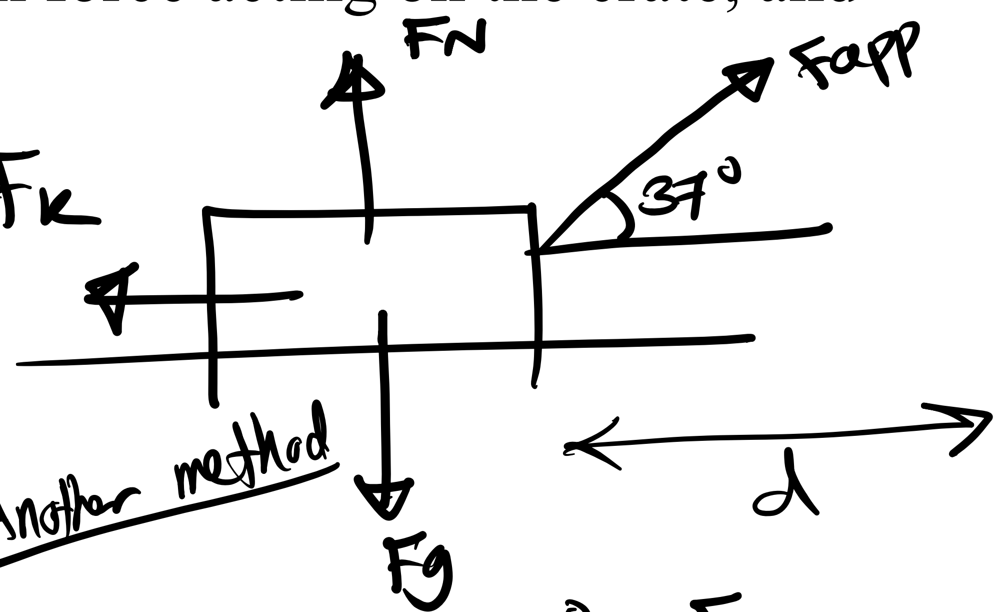
Work is positive ⇒ energy is being transfered into the system

" " negative ⇒ energy is being taken out of the system

" ex : friction "

© 2016 Pearson Education, Ltd.

A person pulls a 50-kg crate 40 m along a horizontal floor by a constant force $F_{app} = 100\ N$ which acts at a 37° angle as shown. The floor is rough and exerts a friction force $F_f = 50\ N$ Determine (a) the work done by each force acting on the crate, and (b) the net work done on the crate.



$W_{app} = F_{app}\, d \cos\theta$

$= 100 \times 40 \times \cos 37°$

$= +3200\ J$

$W_N = 0$

$W_g = 0$

$W_{fr} = F_k\, d \cos\theta$

$= 50 \times 40 \times \cos 180$

$= -2000\ J$

$W_{net} = W_{app} + W_N + W_g + W_{fr}$

$= 1200\ J$

Another method

$F_{net} = F_{app} \cos 37° - F_k$

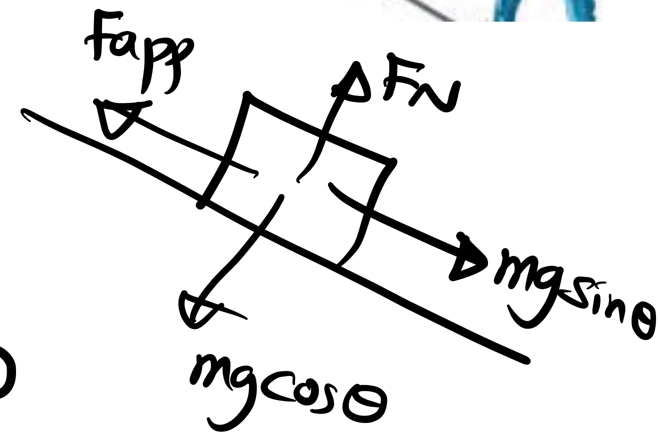
$W_{net} = F_{net}\, d \cos 0$

$= 1200\ J$

© 2016 Pearson Education, Ltd.

$a=0$

A 380-kg piano slides 2.9 m down a 25° incline and is kept from accelerating by a man who is pushing back on it parallel to the incline. Determine: (a) the force exerted by the man, (b) the work done on the piano by the man, (c) the work done on the piano by the force of gravity, and (d) the net work done on the piano. Ignore friction.



(a) $mg\sin\theta - F_{app} = 0$

$F_{app} = mg\sin\theta$

$= (380)(9.8)\sin 25^\circ$

$= 1574\text{ N}$

(b) $W_{app} = F_{app}\, d \cos\theta$

$= (1574)(2.9)\cos 180^\circ$

$= -4565\text{ J}$

(c) $W_g = (mg\sin\theta)(d)\cos 0$

$= +4565\text{ J}$

(d) $W_{net} = 0$

© 2016 Pearson Education, Ltd.

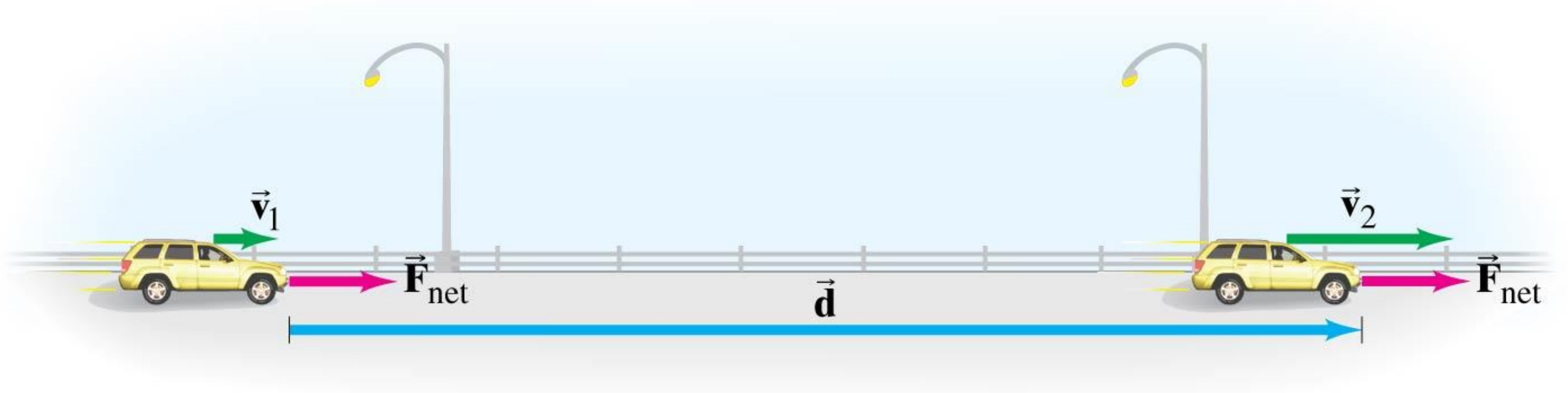
Kinetic Energy and the Work-Energy Principle

- If we write the acceleration in terms of the velocity and the distance, we find that the work done here is

$$W_{\text{net}} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 \qquad (6\text{-}2)$$

- We define the kinetic energy:

$$\boxed{\text{KE} = \tfrac{1}{2}mv^2.} \qquad (6\text{-}3)$$



© 2016 Pearson Education, Ltd.

6-3 Kinetic Energy and the Work-Energy Principle

- This means that the work done is equal to the change in the kinetic energy:

$$W_{\text{net}} = \Delta \text{KE} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2. \qquad (6\text{-}4)$$

- If the net work is positive, the kinetic energy increases.
- If the net work is negative, the kinetic energy decreases.

© 2016 Pearson Education, Ltd.

How much work must be done to stop a 925-kg car traveling at 95 km/h?

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$= \frac{1}{2}(925)(0)^2 - \frac{1}{2}(925)(26.4)^2$$
$$= -3.2 \times 10^5 \text{ J}$$

$95 \div 3.6$
$= 26.4 \text{ m/s}$

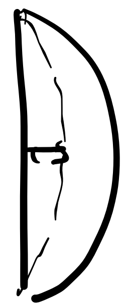
How much work must be done to accelerate a 1000 kg car traveling from rest to 100 km/h?

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$= \frac{1}{2}(1000)(27.7)^2 - 0$$
$$= 3.8 \times 10^5 \text{ J}$$

$100 \div 3.6$
$= 27.7 \text{ m/s}$

© 2016 Pearson Education, Ltd.

An 85-g arrow is fired from a bow whose string exerts an average force of 105 N on the arrow over a distance of 75 cm. What is the speed of the arrow as it leaves the bow?



$$W_{net} = F_{net}\, d \cos 0 = \frac{1}{2} m v_f^2 - \underbrace{\frac{1}{2} m v_i^2}_{\text{zero}}$$

$$(105)(0.75) = \frac{1}{2}(0.085)\, v_f^2$$

$$v_f = 43 \text{ m/s}$$

© 2016 Pearson Education, Ltd.

Potential Energy

An object can have potential energy by virtue of its surroundings.

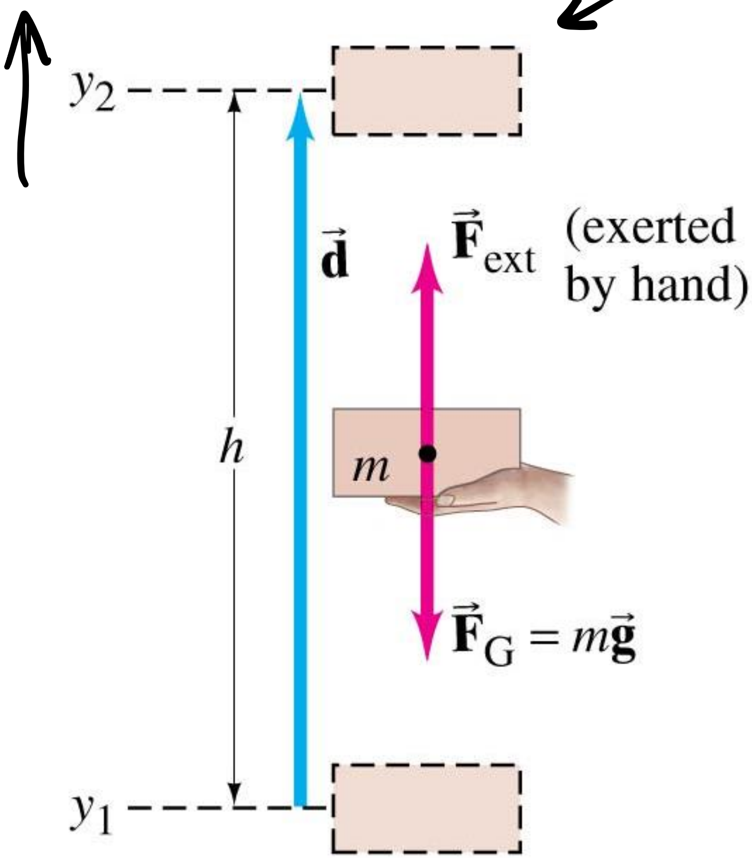
Familiar examples of potential energy:

- A wound-up spring
- A stretched elastic band
- An object at some height above the ground

© 2016 Pearson Education, Ltd.

Potential Energy Work done by external force

the object is moving with constant velocity



In raising a mass m to a height h, the work done by the external force is

$$W_{\text{ext}} = F_{\text{ext}}\, d \cos 0° = mgh \qquad \text{(6-5a)}$$
$$= mg(y_2 - y_1)$$
$$= mgy_2 - mgy_1$$

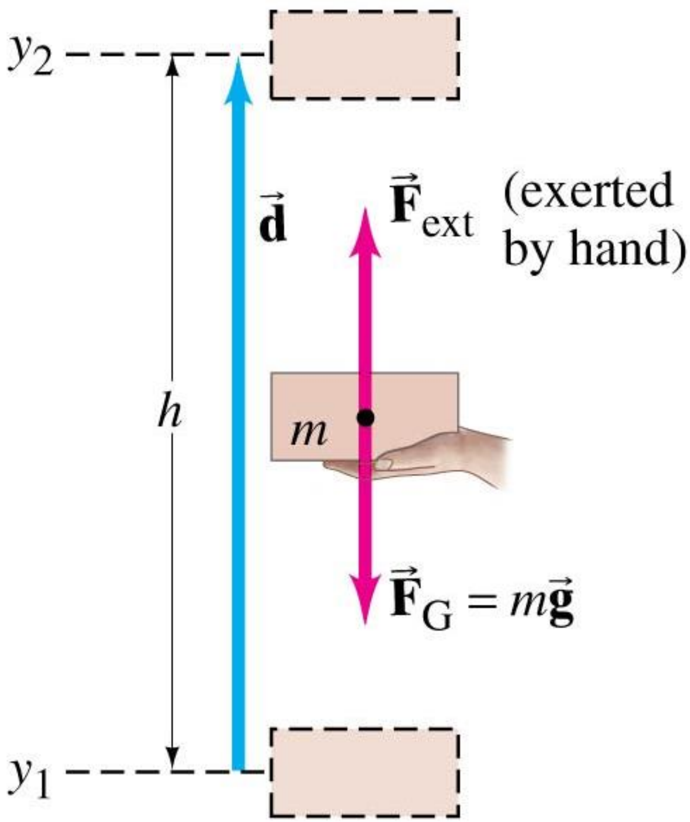
We therefore define the gravitational potential energy:

$$\boxed{PE = mgy} \qquad \text{(6-6)}$$

$$\underline{W_{ext} = PE_f - PE_i}$$
$$= \Delta PE$$

© 2016 Pearson Education, Ltd.

Potential Energy (Work done by gravity)



In raising a mass *m* to a height *h*, the work done by the gravitational force is

$$\mathrm{W}_g = F_g d \cos 180$$
$$= -mg(y_2 - y_1)$$
$$= -mgy_2 + mgy_1$$

$$W_g = PE_i - PE_f$$
$$= -\Delta PE$$

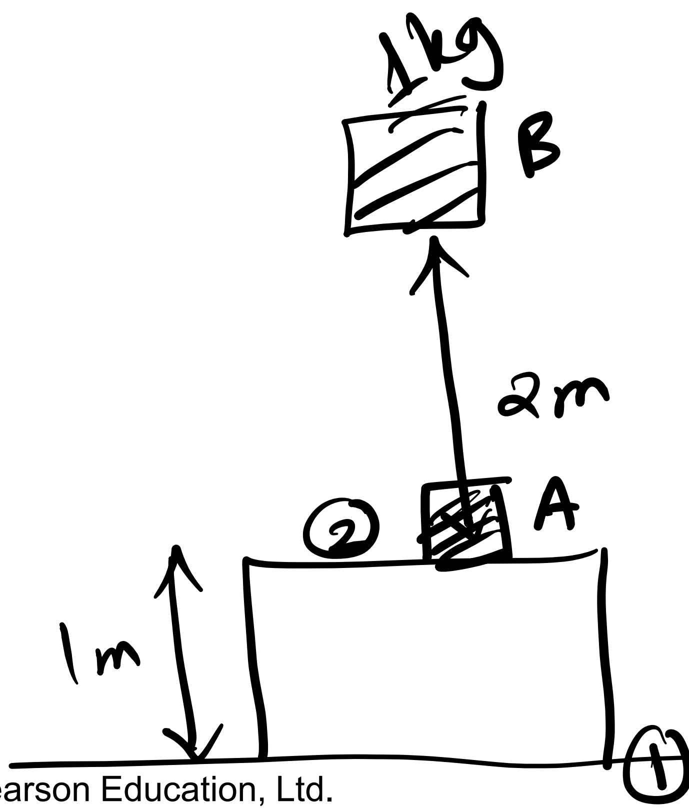
© 2016 Pearson Education, Ltd.

Potential Energy

If $PE = mgy$, where do we measure y from?

It turns out not to matter, as long as we are consistent about where we choose $y = 0$. Only changes in potential energy can be measured.

ΔPE as object moves from A → B



relative to 1

$P.E_i = mg(1)$

$P.E_f = mg(3)$

$\Delta P.E = mg(3-1)$

relative to (2)

$P.E_i = mg(0)$

$P.E_f = mg(2)$

$\Delta P.E = mg(2-0)$

© 2016 Pearson Education, Ltd.

A 1.60-m-tall person lifts a 1.65-kg book off the ground so it is 2.20 m above the ground. What is the potential energy of the book relative to (a) the ground, and (b) the top of the person's head? (c) How is the work done by the person related to the answers in parts (a) and (b)?

relative to the ground

$P.E_i = 0$

$P.E_f = mg(2.2)$

$\simeq 36 J$

$\Delta P.E = 36 - 0$

$= 36 J$

relative to the person's head

$P.E_i = mg(-1.6)$

$= -26 J$

$P.E_f = mg(0.6)$

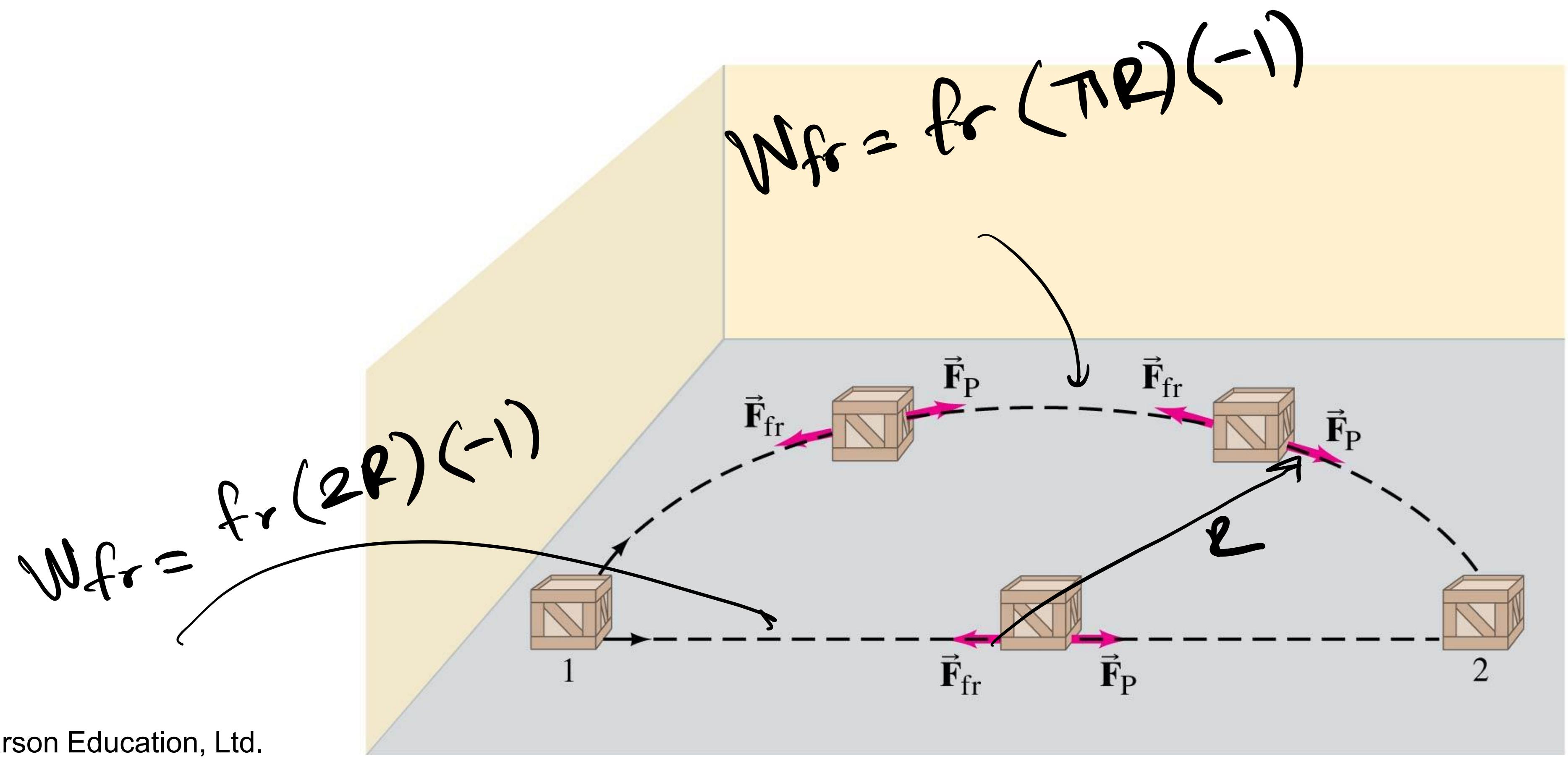
$\simeq 10 J$

$\Delta P.E = 36 J$

© 2016 Pearson Education, Ltd.

Conservative and Nonconservative Forces

If friction is present, the work done depends not only on the starting and ending points, but also on the path taken. Friction is called a nonconservative force.



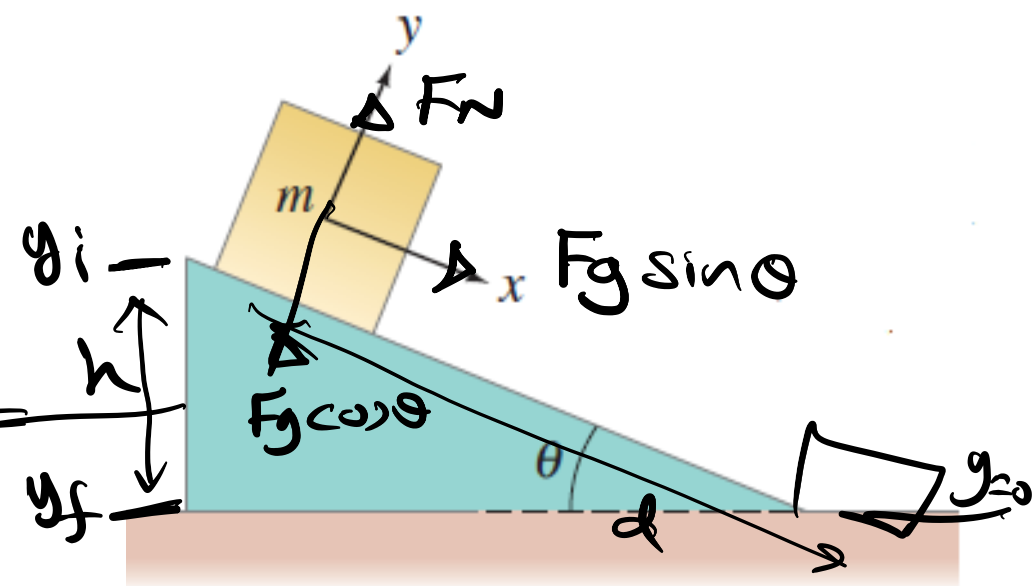
© 2016 Pearson Education, Ltd.

Conservative and Nonconservative Forces

object slides down a smooth surface with a distance (d)

What is W_g?

$$W_g = (mg\sin\theta)(d)\cos 0$$
$$= mgd\sin\theta$$

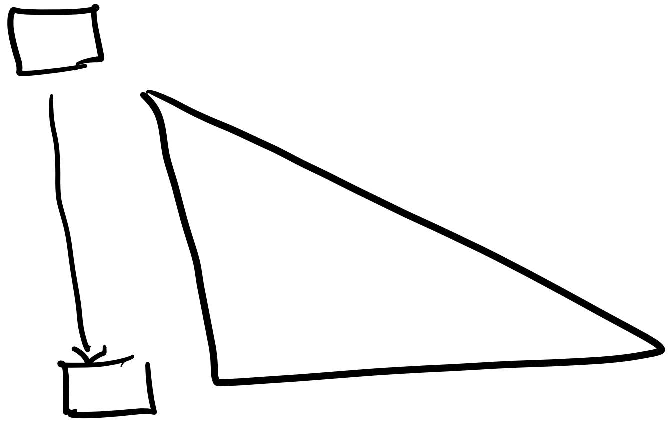


$$W_g = P.E_i - P.E_f$$
$$= mgy_i - mgy_f$$
$$= mg(y_i - y_f) = mgh$$
$$= mgd\sin\theta$$

$$\sin\theta = \frac{h}{d}$$
$$h = d\sin\theta$$

© 2016 Pearson Education, Ltd.

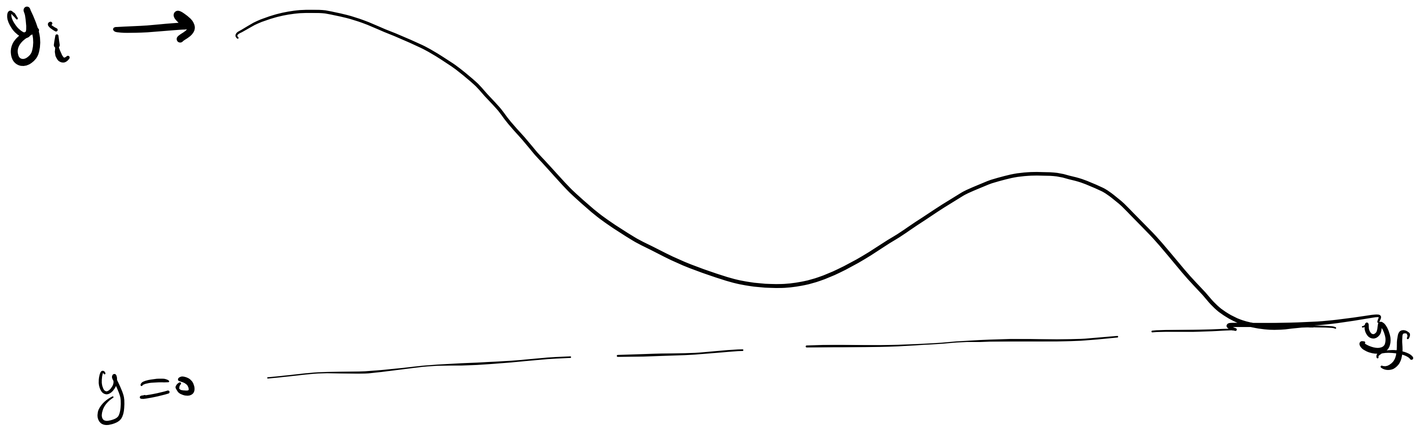
Work done by gravity during the "free fall"



$$W_g = mgh = mgd\sin\theta$$

OR

$$W_g = P.E_i - P.E_f$$
$$= mgh = mgd\sin\theta$$



conservative force

→ Work done does not depend on the actual path travelled by the particle, it depends only on the initial and final points.

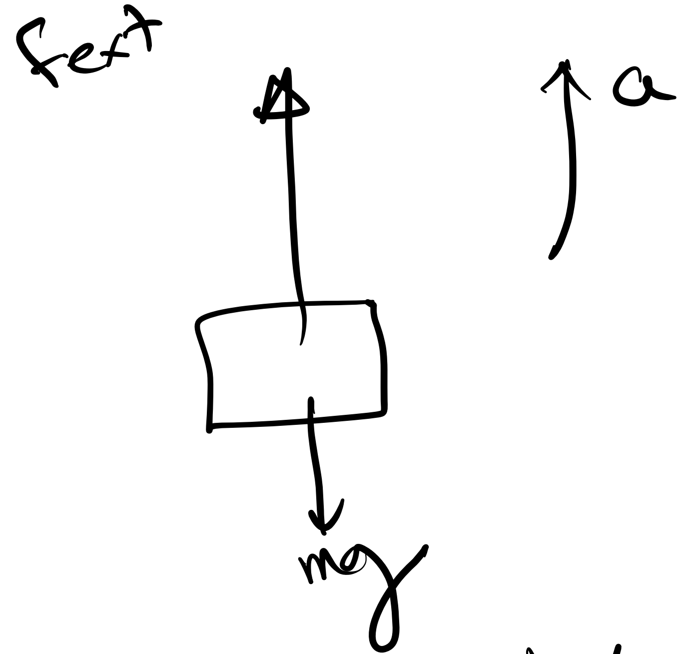
ex : gravitional force

Nonconservative force :

the work done depends on the actual path

ex : friction force

W_{ext} ?



$$W_{net} = W_{ext} + W_g$$

$$\Delta K.E = W_{ext} - \Delta P.E$$

$$W_{ext} = \Delta K.E + \Delta P.E$$

$$W_{NC} = \Delta K.E + \Delta P.E$$

6-5 Conservative and Nonconservative Forces

Therefore, we distinguish between the work done by conservative forces and the work done by nonconservative forces.

We find that the work done by nonconservative forces is equal to the total change in kinetic and potential energies:

$$W_{\mathrm{NC}} = \Delta \mathrm{KE} + \Delta \mathrm{PE} \qquad (6\text{-}10)$$

$$W_{NC} = \left(\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2\right) + \left(mgy_2 - mgy_1\right)$$

© 2016 Pearson Education, Ltd.

$W_{NC} = \Delta K.E + \Delta P.E$

if there are no nonconservative forces.

$W_{NC} = 0$

$0 = \Delta K.E + \Delta P.E$

$0 = (\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2)$
$+ (mgy_2 - mgy_1)$

$$\boxed{mgy_i + \frac{1}{2}mv_i^2 = mgy_f + \frac{1}{2}mv_f^2}$$

E = mechanical energy

$= K.E + P.E$

$E = \frac{1}{2}mv^2 + mgy$

6-6 Mechanical Energy and Its Conservation

If there are no nonconservative forces, the sum of the changes in the kinetic energy and in the potential energy is zero—the kinetic and potential energy changes are equal but opposite in sign.

This allows us to define the total mechanical energy:

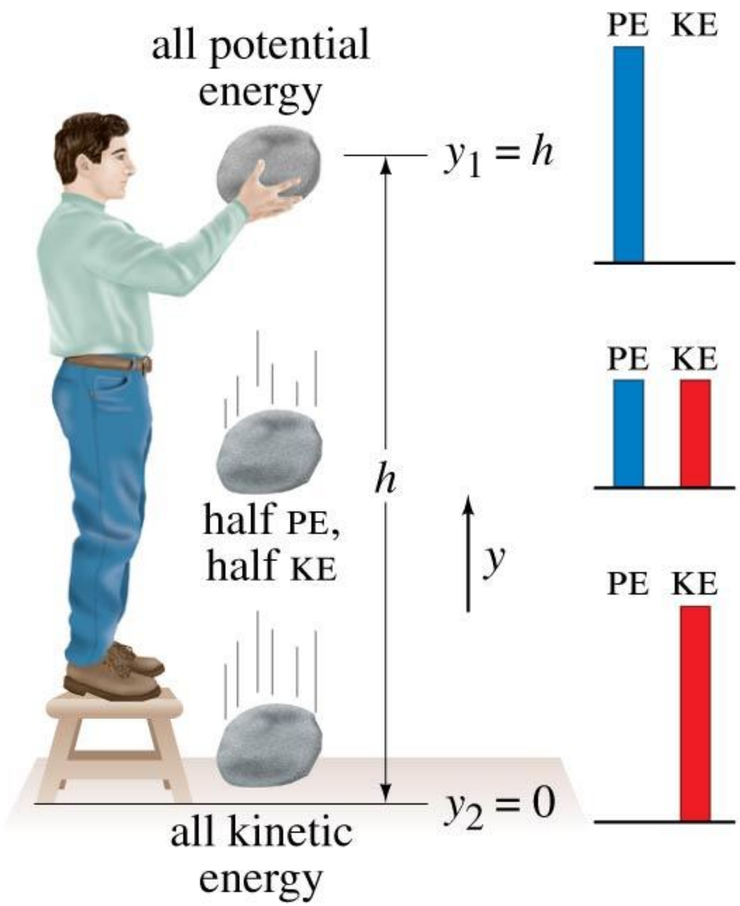
$$E = \mathrm{KE} + \mathrm{PE}$$

And its conservation:

$$\boxed{E_2 = E_1 = \text{constant.}} \qquad \text{(6-12b)}$$

© 2016 Pearson Education, Ltd.

6-7 Problem Solving Using Conservation of Mechanical Energy



In the image on the left, the total mechanical energy is:

$$E = \text{KE} + \text{U} = \tfrac{1}{2}\, mv^2 + mgy$$

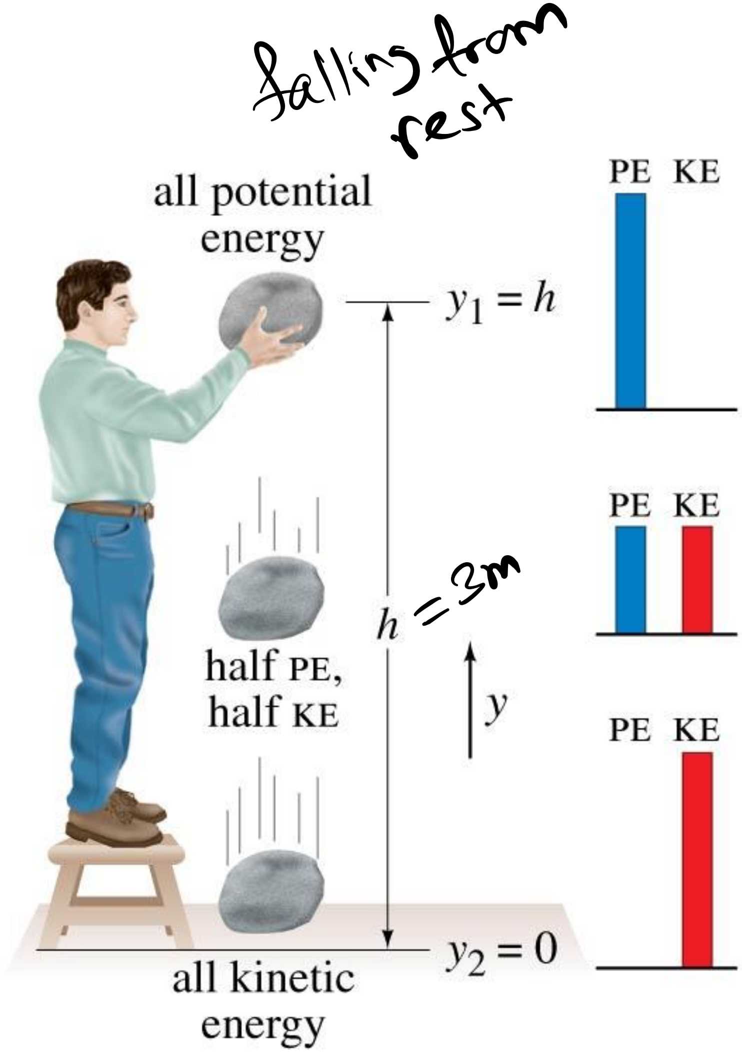
potential energy

The mechanical energy is constant

$$\boxed{\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2}$$

o friction

© 2016 Pearson Education, Ltd.



calculate the rock's velocity when it has fallen to 1.0 m above the ground.

$$\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f$$

$$0 + (9.8)(3) = \frac{1}{2}v_f^2 + (9.8)(1)$$

$$v_f = \sqrt{39.2}$$

$$v_f = 6.3 \text{ m/s}$$

© 2016 Pearson Education, Ltd.

Assuming the height of the hill is 40 m, and the roller-coaster car starts from rest at the top, calculate (a) the speed of the roller-coaster car at the bottom of the hill, and (b) at what height it will have half this speed.

(a) $\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f$

$0 + (9.8)(40) = \frac{1}{2}v_f^2 + 0$

$v_f = 28$ m/s

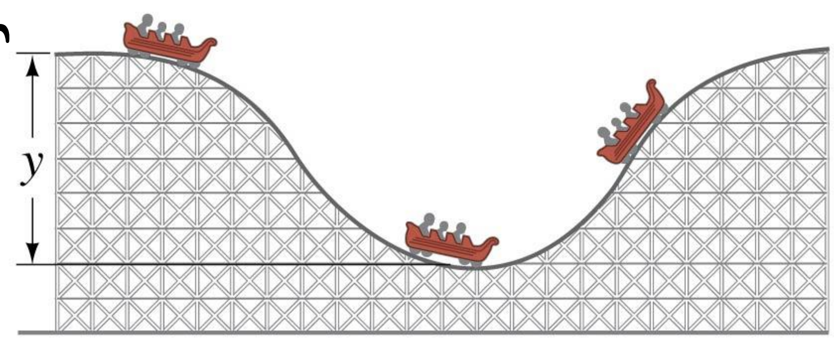
(b) $\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f$

$0 + (9.8)(40) = \frac{1}{2}(14)^2 + (9.8)(y_f)$

$y_f = 30$ m

i

$y = 40$m



$y = 0$

© 2016 Pearson Education, Ltd.

Other Forms of Energy and Energy Transformations; the Law of Conservation of Energy

Some other forms of energy:

- Electric energy, nuclear energy, thermal energy, chemical energy.

Work is done when energy is transferred from one object to another.

Accounting for all forms of energy, we find that the total energy neither increases nor decreases. Energy as a whole is conserved.

© 2016 Pearson Education, Ltd.

6-9 Energy Conservation with Dissipative Processes; Solving Problems

$W_{NC} = \Delta K.E + \Delta P.E$

$W_{NC} = (KE_2 - KE_1) + (PE_2 - PE_1)$

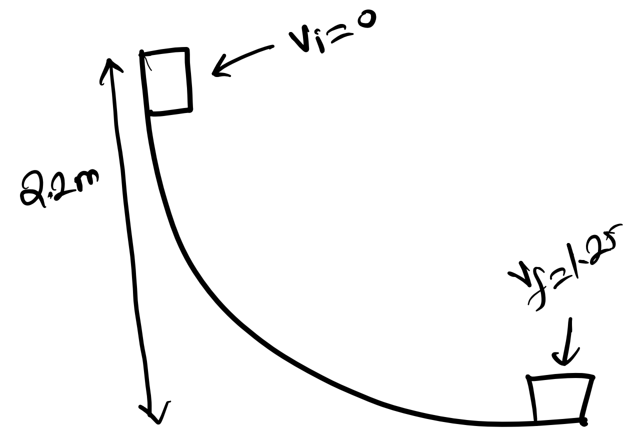
$$\mathrm{KE}_1 + \mathrm{PE}_1 + W_{\mathrm{NC}} = \mathrm{KE}_2 + \mathrm{PE}_2 .$$

$$\boxed{\tfrac{1}{2}mv_1^2 + mgy_1 - F_{\mathrm{fr}}d = \tfrac{1}{2}mv_2^2 + mgy_2}$$

© 2016 Pearson Education, Ltd.

A 16.0-kg child descends a slide 2.20 m high and, starting from rest, reaches the bottom with a speed of 1.25 m/s . How much thermal energy due to friction was generated in this process?

$$\frac{1}{2}mv_i^2 + mgy_i + W_{fr} = \frac{1}{2}mv_f^2 + mgy_f$$



$$\frac{1}{2}(16)(0)^2 + (16)(9.8)(2.2) + W_{fr} = \frac{1}{2}(16)(1.25)^2 + 0$$

$$W_{fr} = -332\text{J}$$

$$E_{Th} = 332\text{J}$$

© 2016 Pearson Education, Ltd.

a 5.0 kg object falls down from rest from a height of 10 m reaching the ground with speed of 12 m/s, calculate the work done by air resistance?

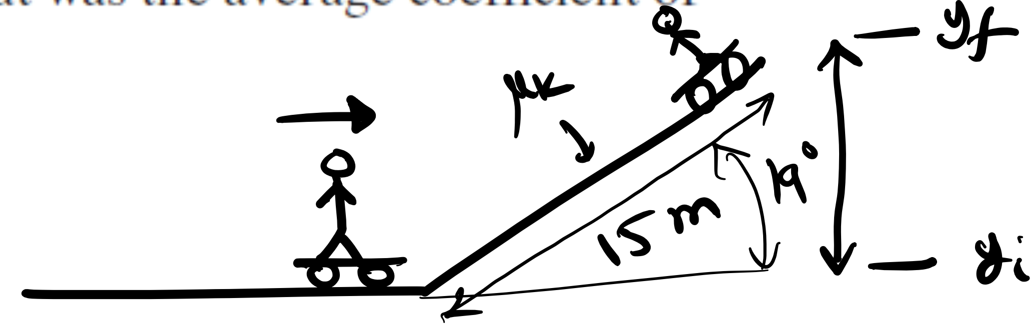
$$\frac{1}{2}mv_i^2 + mgy_i + W_{air} = \frac{1}{2}mv_f^2 + mgy_f$$

$$0 + (5)(9.8)(10) + W_{air} = \frac{1}{2}(5)(12)^2 + (5)(9.8)(0)$$

$$W_{air} = -130\,J$$

44. (II) A skier traveling 11.0 m/s reaches the foot of a steady upward 19° incline and glides 15 m up along this slope before coming to rest. What was the average coefficient of friction?

$f_k = \mu mg\cos\theta$



$y_f = 15\sin 19°$

$= 4.9$

$$\frac{1}{2}mv_i^2 + mgy_i - f_k d = \frac{1}{2}mv_f^2 + mgy_f$$

$$\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i - \mu_k \cancel{m}gd\cos\theta = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f$$

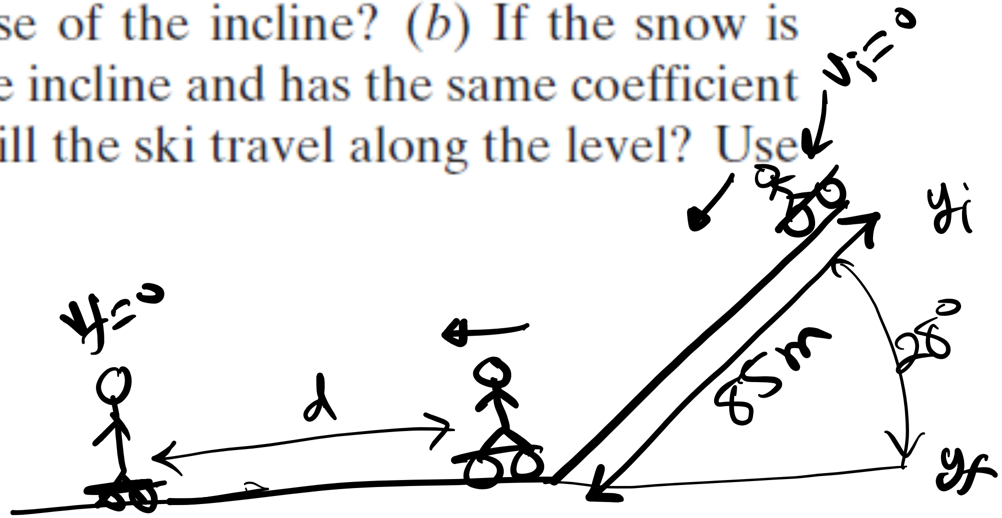
$$\frac{1}{2}(11)^2 + 0 - \mu_k(9.8)(15)\cos 19° = 0 + (9.8)(4.9)$$

$$\mu_k = 0.09$$

© 2016 Pearson Education, Ltd.

42. (II) A ski starts from rest and slides down a 28° incline 85 m long. (*a*) If the coefficient of friction is 0.090, what is the ski's speed at the base of the incline? (*b*) If the snow is level at the foot of the incline and has the same coefficient of friction, how far will the ski travel along the level? Use energy methods.

$y_i = 85 \sin 28$
$= 40\text{m}$



(a) $\frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i - \mu_k \cancel{m}g\cos\theta d = \frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f$

$0 + (9.8)(40) - 0.09(9.8)\cos 28(85) = \frac{1}{2}v_f^2 + 0$

$v_f = 25.5$ m/s

(b) $\frac{1}{2}\cancel{m}v_i^2 + \cancel{mgy_i} - \mu_k \cancel{m}gd = \frac{1}{2}\cancel{m}v_f^2 + \cancel{mgy_f}$

$\frac{1}{2}(25.5)^2 - (0.09)(9.8)d = 0$

$d = \underline{368\text{ m}}$

© 2016 Pearson Education, Ltd.

6-10 Power

Power is the rate at which work is done—

$$\overline{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}} \quad (6\text{-}17)$$

© 2016 Pearson Education, Ltd.

In the SI system, the units of power are watts:

$$1\text{W} = 1 \text{ J/s}$$

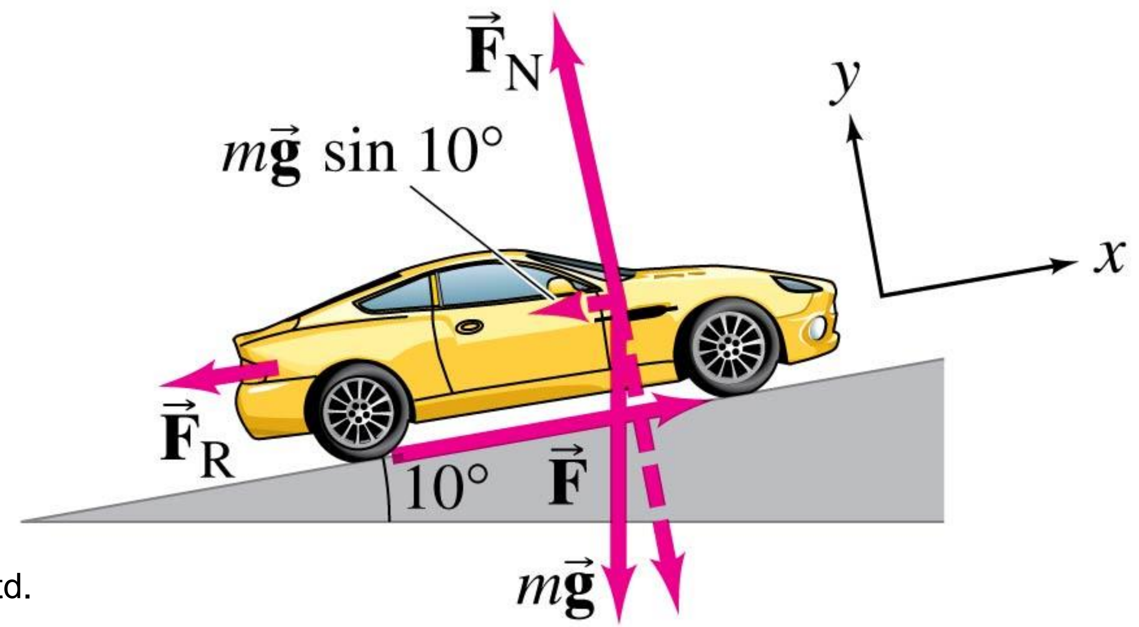
The difference between walking and running up these stairs is power—the change in gravitational potential energy is the same.

6-10 Power

Power is also needed for acceleration and for moving against the force of gravity.

The average power can be written in terms of the force and the average velocity:

$$\overline{P} = \frac{W}{t} = \frac{Fd}{t} = F\overline{v}. \qquad (6\text{-}18)$$



© 2016 Pearson Education, Ltd.

49. (I) How long will it take a 2750-W motor to lift a 385-kg piano to a sixth-story window 16.0 m above?

$$W_{ext} = mgh$$

$$W_{ext} = (385)(9.8)(16)$$

$$= 60370\text{ J}$$

$$P = \frac{W}{t}$$

$$t = \frac{W}{P} = \frac{60370}{2750} \simeq 22\text{ s}$$

© 2016 Pearson Education, Ltd.

56. (II) A 975-kg sports car accelerates from rest to 95 km/h in 6.4 s. What is the average power delivered by the engine?

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

26.4 m/s

$$= \frac{1}{2}(975)(26.4)^2 - 0$$

$$= 339770 \text{ J}$$

$$P = \frac{W}{6.4\text{ s}} = 53090 \text{ W}$$

$$1 \text{ hp} = 746 \text{ W}$$

$$P = \frac{53090}{746} \simeq 71 \text{ hp}$$

© 2016 Pearson Education, Ltd.

Summary of Chapter 6

- Work: $W = Fd \cos \theta$
- Kinetic energy is energy of motion: $KE = \frac{1}{2} mv^2$
- Potential energy is energy associated with forces that depend on the position or configuration of objects.
- The net work done on an object equals the change in its kinetic energy.
- If only conservative forces are acting, mechanical energy is conserved.
- Power is the rate at which work is done.

© 2016 Pearson Education, Ltd.